Long-term debt and hidden borrowing*

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Abstract

Borrowers can raise funds from a competitive banking sector that shares information, and opaque hidden lenders. Hidden lenders allow borrowers to conceal poor results, and thereby affect contracts in the banking sector. In equilibrium, borrowers obtain funds from both sectors simultaneously. The lack of transparency generates cross-subsidies between different borrowers who are observationally equivalent to banks and face the same interest rate. As the cost of hidden borrowing falls, an increasing number of borrowers face identical terms; for sufficiently low costs, all borrowers who take loans (which may include inefficient borrowers) use the same bank debt contract.

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1 Introduction

Firms and households have access to various sources of borrowing. The differences in seniority, covenants, and interest rates may induce an apparent "pecking order" among loans. However, loans also differ in the extent of their transparency to other lenders. Whereas some lenders perfectly share information—through a public credit registry, for example—other lenders may not engage in such information sharing. Thus, borrowers may choose more opaque loans to conceal information from others.

This paper investigates the effects of the presence of opaque loans on formal bank loans. In particular, we explore the implications for the types of loans offered and adopted, liquidation decisions, and welfare. We argue that the presence of opaque lenders limits the contracting options of other lenders: If all lenders perfectly share information, loans induce borrowers to reveal their solvency at all times through interest rates that are responsive to repayment schedules. However, if borrowers can secretly obtain funds, loan repayments might reflect not only a borrower's creditworthiness but also her access to alternative loans. For this reason, loans become less responsive to interim payments. In addition, we demonstrate that borrowers may simultaneously access both opaque and transparent loans even though the interest rate on the more-opaque loans may be higher than that on transparent loans. We also demonstrate that different types of borrowers—i.e., with different abilities to repay—might appear indistinguishable to the formal banking sector and thus face identical borrowing terms. Cross-subsidies between borrowers imply that welfare can diminish with the availability and affordability of hidden loans as a result of both inefficient liquidation and borrowers accessing relatively expensive opaque funds.

Our results provide one explanation for the empirical observation that borrowers obtain loans from apparently costly lenders without fully exhausting less expensive sources. Firms, for example, may use costly trade credit and personal loans from the owner before exhausting their credit lines and while having free collateral.[1] Similarly, households may use expensive and opaque forms of borrowing, such as payday lending, to cover mortgage repayments even though they have positive housing equity.

Missing a repayment can trigger revised terms with the bank and lead to a higher future interest rate, reflecting the bank's renewed assessment of the borrower's ability to repay. Approaching the bank to revise the terms of a loan may be costly to the borrower because

[1] For example, in the 1998 National Survey of Small Business Finance (NSSBF), among the firms with bank debt not exceeding the value of their land (a conservative estimate of firms with free collateral), 14.7% used trade credit and 13.5% used lines of credit.

it would reveal information about current and future cash flows. To avoid this penalty, an entrepreneur might borrow from elsewhere, taking a personal loan, for example, to conceal the bad news that the enterprise has suffered a negative shock. In turn, this behavior makes missing a payment even worse news, as it reflects a negative shock so large that it is prohibitively costly to conceal.

We illustrate the interaction between publicly observable and hidden borrowing more formally. We introduce a two-period model in which agents have access to an investment project that yields cash flows correlated across time. Borrowers can fund the project through two sources: a competitive banking sector that shares information and an opaque, and more expensive, lending sector that does not. Banks are senior claimants and seek to obtain information regarding borrowers through interim payments.

If the alternative source of borrowing is sufficiently expensive (or absent), banking contracts will achieve a first-best outcome. By rewarding higher interim payments with lower future interest rates, the optimal contract provides borrowers with incentives to fully reveal their intermediate cash flows. However, with a viable alternative hidden lender, a borrower might be tempted to borrow from that source to disguise her type. The original lender in the banking sector anticipates this possibility. In general, this possibility will lead to a more limited set of repayment schedules in the optimal contract. Further, we demonstrate that borrowers borrow from the opaque sector to make interim repayments. Thus, in equilibrium, borrowers simultaneously borrow from both the formal banking sector and the opaque sector of hidden lenders. This behavior is a well-documented phenomenon and, in our model, is not the result of behavioral biases.

By imposing a distributional assumption on borrower types, we fully characterize the contract and its repayment schedules. We demonstrate that as the cost of hidden borrowing falls, the equilibrium changes from a continuum of contracts that fully separate borrowers to a countable set of contracts, each attracting a pool of borrowers. As the cost of hidden borrowing falls, borrowers end up in larger pools on average. Eventually, the banking sector offers only a single pooling contract.

We perform comparative statics exercises that lead to some empirical predictions. We find that more expensive hidden lending improves the sorting of borrowers by banks. This improved sorting allows for a greater variety of lending arrangements in the formal banking sector, as different types of borrowers are more likely to face different borrowing terms. When hidden borrowing is sufficiently inexpensive, all borrowers face identical terms in the banking sector. Thus, one could think that the sophistication of the banking sector depends on the cost of hidden lending. The effect of increasing the cost of hidden lending may be similar to making it more transparent; thus, technological and regulatory changes that improve information sharing should have similar effects.[2] Furthermore, more expensive hidden lending leads to improved terms (that is, lower interest rates) in the formal sector because it is then more expensive for borrowers to conceal their creditworthiness. In the presence of hidden lending, borrowers may liquidate projects too seldom, as there are cross-subsidies induced by concealment. Increasing the cost of borrowing from hidden sources increases welfare over the range where more than one contract is adopted, and one might naturally suppose that this increase in welfare also increases the volume of loans initiated. Thus, influencing and regulating obscure or opaque sources of funds can help to improve flexibility and credit conditions in the more formal banking sectors.

Related Literature and Supportive Evidence The historical and international evidence on information sharing in the financial sector is consistent with the predictions of our model (see, for example, the edited volume Miller, 2003; Hunt, 2006; Jentzsch, 2007; and Jappelli and Pagano, 2006). In particular, Brown, Jappelli, and Pagano (2009), in an investigation of firms in Eastern Europe and the former Soviet Union, conclude that information sharing is associated with improved availability and lower costs of credit to firms (p. 1). Cowen and De Gregorio (2003) present evidence that in Chile, information sharing led to an increase in the volume of lending. There are alternative explanations for these findings;[3] however, existing literature on the complexity of contracts is lacking. Our model suggests that increased availability of creditor information should lead to debt contracts with more flexible repayment terms and schedules rather than, for example, fixed repayment levels at fixed dates. This expectation appears to be consistent with anecdotal evidence, for example, in the U.K., where there has been growth in flexible (or lifestyle) mortgages at a time when consumer credit scoring has expanded. In contrast, at the firm level, the empirical accounting literature has recently focused on the increasing use of financial innovations, such as off-balance-sheet lease financing, as a form of opaque borrowing (Cornaggia et al., 2012; Zechman, 2010). Such opaque borrowing has contributed to making the analysis of balance sheets by creditors and rating agencies less precise (Cornaggia et. al., 2013).

[2] For an example of partially hidden lending, where greater transparency is equivalent to higher interest rates in the opaque sector, see Bar-Isaac and Cuñat, 2012.

[3] See Padilla and Pagano, 1997; Jappelli and Pagano, 1993; and, more broadly, the discussion in Jappelli and Pagano, 2006, for alternative but related models of information sharing.

In this paper, the banking sector cannot write contracts in which payments depend on amounts borrowed from hidden lenders, a natural consequence of the assumption that the banking sector cannot observe this borrowing. This paper is, therefore, related to a growing literature on non-exclusive contracts and hidden savings.

Our focus on different lending sectors that vary in the information that they possess, as well as the simple comparative statics analysis that this focus enables, distinguishes our paper from the literature on exclusivity. For example, there are models of non-exclusivity with simultaneous contracting (Bisin and Guaitoli, 2004; Jaynes, 1978; and Arnott and Stiglitz (1991) in the context of insurance markets), with sequential access to loans (Bizer and DeMarzo (1992)) and with financial intermediaries who are *ex-ante* identical.

In the optimal contracts that we characterize, interim payments provide useful information that can allow for more-efficient outcomes. This result mirrors observations in Allen (1985) and Dionne and Lasserre (1985). Thus, hidden borrowing or savings (as in Cole and Kocherlakota, 2001) can create inefficiency in these environments by reducing the information provided by interim payments.

A feature of our analysis is that we vary the cost of borrowing from the hidden source. Allen (1985) and others focus on the case in which this cost is equal to the social planner's rate.[4] To generate monotonicity in repayment schedules, Innes (1990) considers the case in which money can be repaid immediately such that the cost of borrowing is essentially zero.

Finally, a key element of the model is that a lender may not perfectly observe all of the loans that a borrower may hold. Empirically, this situation is certainly true. For example, although information sharing takes place through credit bureaus, many lenders choose neither to pay for access to such information nor to provide it. Trade credit, informal black-market lending, and personal loans to entrepreneurs subsequently used in their firms are clear examples of potentially hidden loans. Further examples include consumer credit, store credit, payday loans, and other sources that do not cooperate with organized information-gathering credit bureaus, both in developing countries and elsewhere, both currently and historically.[5] For instance, Barron and Staten (2003) highlight that in some Latin American

[4]The general model of Doepke and Townsend (2004), as illustrated in their example in Section 7.1, allows for this more general interest rate; however, as in Cole and Kocherlakota (2001) and Ljungqvist and Sargent (2003), they consider hidden saving and insurance rather than hidden borrowing, and they focus on numerical rather than analytical solutions.

[5]For example, in the U.S., payday lenders do not share information with banks (Elliehausen and Lawrence, 2001; Mann and Hawkins, 2007). However, it has been shown that their presence alters bor-

countries, there are "comprehensive credit histories on consumers but only on loans held by commercial banks"(pp. 273-4). Furthermore, even when a lender has access to a credit bureau, the costs associated with accessing and processing the relevant information may lead lenders to obtain and use this information only in particular circumstances. Such circumstances would include the loan-approval stage, missed payments, and renegotiation; otherwise, continual monitoring is unlikely. In this paper, we simply assume that some types of borrowing are not commonly observed by all lenders.

2 The Model

Although the underlying economic mechanisms have wider applicability, we focus the model on the particular example of a small business that is raising funds for a capital-investment project that will generate an interim and final return. Because these payouts are positively correlated, there is additional information at the interim stage that is useful for assessing creditworthiness, which is heterogeneous. The firm has access to both a competitive banking sector and hidden lender. One can think of the hidden lender as a personal loan to the entrepreneur secretly diverted to the firm.

We introduce a two-period model to consider the interaction between alternative sources of borrowing: a transparent banking sector and an opaque hidden lender (or lending sector).

2.1 Lending Sectors

In the transparent sector, credit is provided by a continuum of agents that we call banks. Banks are risk-neutral deep pockets, and there is competition among them. Banks share information, so the borrowing position of any borrower from a bank is perfectly observable and verifiable among all banks. We normalize the gross riskless market interest rate of this banking sector to one. Our focus is on debt, and we allow banks to offer only debt contracts. This situation matches many practical applications. Moreover, there is an extensive literature that seeks to explain the prevalence of debt contracts as a response to informational or incentive problems that are orthogonal to the problem that we consider (for example, Jensen and Meckling, 1976; Townsend, 1979; Innes, 1990; Gale and Hellwig, 1985; among many others).

The key assumptions concerning the banking sector are that it is competitive and shares information. More formally, we assume that banks compete in two stages: (i) in initiating

rowers' payments of other loans. In particular, mortgage delinquency following aggregate liquidity shock is significantly lower in areas where there are payday lenders (Morse, 2011).

the loan and (ii) at an interim stage following the first repayment. To make competition at the interim stage relevant, we assume that after the first repayment, refinancing the loan at the interim stage (that is, repaying the full amount borrowed or repaying whatever is left following the first repayment and taking out another loan from another bank) entails no penalty to the borrower.[6] We also assume that banks perfectly share the information about the borrower's payments and outstanding loans. In particular, this assumption implies that they cannot simply replicate hidden lending, as they have no means to hide such contracts from other banks.

Whereas we restrict banks to offering only debt contracts, we allow each bank to offer a contract with as many repayment schedules as it wishes during the initial period. Formally, this situation is equivalent to banks competing with each other in the first stage by making offers that consist of different second period repayment options that depend on interim payments. We denote the contract as $\{p, q(p)\}$, where each schedule within the contract is defined by a first-period repayment, p, and associated second-period repayment, $q(p)$, where both p and $q(p)$ are non-negative. Given that banks offer debt contracts, each first-period payment p has an associated outstanding balance at $t = 1$, which combined with the second period promised repayment $q(p)$ implies a second-period interest rate. Therefore, we can interpret bank contracts as debt contracts with different interim repayment options, and second period interest rates that depend on the outstanding balance after the interim repayment.

At each stage, both when the loan is initiated and following the first repayment, banks compete sequentially. In the first stage, when the loan is initiated, a bank is picked at random to make an offer. If it does not make an offer, the process ends and the borrower receives no loan. If the borrower accepts the bank's offer, an alternative bank is selected at random to observe the borrower's current contract and make a second offer. If the borrower, after observing both offers, accepts the first bank's offer, no further offers are made. Otherwise, the borrower accepts the second offer and a third bank makes an offer. If this is the case, the first offer ceases to be available. If the borrower, observing the offers of the second and third banks, accepts the second bank's offer, no further offers are made. Otherwise, the borrower accepts the third offer, loses the possibility to accept the second

[6] The opting-out assumption appears empirically reasonable in many markets for long-term debt. Mortgages, for example, have small or no penalties for early payment (see Green and Wachter, 2005). Lines of credit are often used for long-term borrowing and are also fully pre-payable with no penalty. Bonds are often callable (see Sundaresan, 2009).

offer and another bank makes an offer. The sequence continues until the borrower rejects the next alternative offer at any stage.

Competition at the second interim stage is similar, except that initially, the borrower has chosen an existing repayment schedule $p, q(p)$ against which future offers may be compared. We also restrict banks to making simple offers, consisting of single interim and final repayments, to each existing borrower. However, we suppose that when making an offer, a bank can observe the contract offered at $t = 0$ and the repayment schedule of the contract that the borrower has chosen.[7] That is, in the second stage, after observing the contract $\{p, q(p)\}$ held at $t = 0$ and the particular repayment schedule—p and associated $q(p)$—that any particular borrower has adopted, a rival bank competes by making an offer of first- and second-period payments $p'(p, q(p), \{p, q(p)\})$ and $q'(p, q(p), \{p, q(p)\})$, which, as the notation suggests, depend on the initial contract and the option chosen.[8] As before, if the borrower rejects this new offer, the process ends. Otherwise, a new bank can make an offer that may depend on the offer made by the previous bank in addition to the initial contract. Again, the process continues in an open-ended fashion. Either the borrower rejects the new offer and the process ends or the borrower accepts the new offer and a new competing bank offers a simple contract after observing the original contract offered, the schedule that was chosen from that contract, and the last alternative offer taken.[9] We assume that banks will not make offers that can be loss making in expectation at any stage of competition. That is, we assume that the banks will not make offers that would be loss making if the game ended immediately after the offer, so that no further offers are possible.[10]

Within this structure of competition we assume the following lexicographic assumption on bank preferences: Every bank maximizes profits, and, further, if there are no profitable opportunities, it prefers to make an offer that would retain at least one borrower type to

[7] We have in mind that the borrower makes the interim repayment to the bank and that this payment gets reported, but the delay between bank offers is infinitesimal so that the borrower can repay the hidden borrower (if needed) immediately and at no cost.

[8] Note that we do not allow banks to offer more than one schedule at t=1. Instead, a bank competes by offering a simple single alternative contract, (p', q').

[9] This model of competition, where rival banks compete on simple contracts and where competition ends if a borrower rejects an offer, keeps the analysis tractable. In particular, when borrowers know their types, they can only reject offers (which ends competition) or accept them, thus limiting the scope for signaling through the dynamics of the competitive process.

[10] This assumption rules out that a bank makes an offer that is initially making losses, with the expectation that further offers will make it profitable.

an offer that attracts no borrowers.[11]

This structure of competition ensures that borrowers are always able to opt out of their current loan if another bank is willing to offer a more attractive loan.[12] The nature of competition between banks plays an important role in ensuring that bank loans involve no observable cross-subsidies, as discussed in Proposition 1, and that banks earn zero profits. These are reasonable properties of an environment in which banks share information and compete with each other.

Note that if banks could lock borrowers into long-term contracts with no opportunity to renegotiate, then information sharing would be irrelevant in this model. The assumption that banks compete at each stage and that loans can be refinanced without penalty eliminates this possibility.

In addition to the transparent banking sector, we introduce an alternative opaque, or hidden, lending sector that is less efficient than the banking sector and lends at a repayment rate $r > \frac{1}{\nu}$, where ν is a parameter discussed in Section 2.2.[13]

A key feature of this alternative borrowing source is that it does not share information with the rest of the financial system. That is, banks do not observe the borrowing position of any borrower in the opaque sector. Furthermore, we model the opaque sector as a junior lender, which is certainly consistent with the interpretation of a hidden loan as a concealed loan from the firm owner to the firm.[14] In our model, lenders exogenously belong to either the banking sector or the opaque sector.

2.2 Borrowers

Demand for funds comes from borrowers who require funds for an investment project and who are heterogeneous in the quality of their projects. They are risk-neutral and maximize total consumption across periods. If a borrower is indifferent between two contracts, we

[11] Without the assumption that a bank prefers to make a zero-profit offer that retains borrowers to making an offer that makes zero profits but retains no borrowers we cannot rule out equilibria in the sequential Bertrand game that allow banks to make strictly positive profits. Without this assumption, there can be equilibria where an initial bank makes an offer that makes strictly positive profits and no other banks make reasonable offers that earn strictly positive profits since they anticipate that these profits would be competed away; moreover, since they are indifferent between zero profit offers they may choose ones with unreasonable terms that attract no borrowers.

[12] Bennardo et al. (2009) use a similar form of bank competition.

[13] If hidden lenders were as efficient as the banking sector, then $\frac{1}{\nu}$ would be the break-even rate, as discussed below.

[14] In terms of seniority, it is also consistent with trade credit, for example. Other types of hidden lending, including black-market lending, may be more ambiguous with respect to seniority.

suppose that she keeps her existing contract rather than switching to a new bank. If a borrower is picking different repayment schedules from a contract and is indifferent between two repayment schedules, we assume that she picks a contract through the following lexicographic rule: first, she takes up a contract that involves less hidden borrowing, and second, she takes up a contract with higher expected consumption in the final period.

The timing of the model is as follows:

At $t = 0$, each borrower does not know her type. To raise D units of funding necessary to invest in the project, the borrower can choose among the available offers of first- and associated second-period debt-repayment schedules $\{(p, q(p))\}$.

At $t = \frac{1}{2}$, each borrower privately learns the type of her project, which is parameterized by α, where α is continuously distributed on $[0, 1]$. At this point, the borrower can either liquidate the project for D at no cost and fully repay the loan, or can continue with the project and choose a repayment schedule.[15]

At $t = 1$, a borrower realizes a cash flow α that corresponds to her type. At this stage, the borrower may also switch to a competing bank offer $(\widetilde{p}, \widetilde{q})$, where the nature of competition is as outlined above. She can also choose to borrow funds, d_{op}, from the opaque lending source that is hidden from the banks. A loan of the opaque lender is junior to the bank loan, and banks do not observe it. The borrower can use these funds either to consume or to choose one of the repayment schedules from the contract and repay p to the bank.

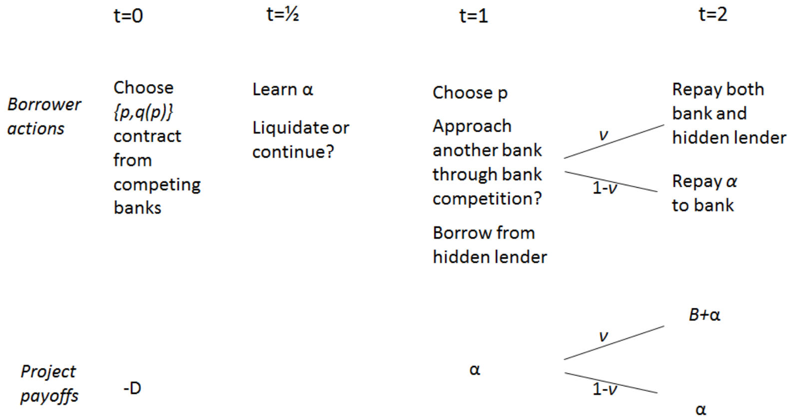
At $t = 2$, the project is successful and delivers $B+\alpha$ with probability ν. Otherwise, the project fails and delivers only α. In both cases, seniority of debt is such that the borrower repays $q(p)$ to the bank first and then repays opaque lenders up to rd_{op}. The borrower consumes all remaining funds.

The parameter α represents the creditworthiness of the borrower, as the expected final cash flow of the project is positively correlated with its interim cash flow. Note that, overall, a project of type α generates a net present value of $-D+\alpha+\nu(B+\alpha)+(1-\nu)\alpha = -D+\nu B+2\alpha$. In particular, the best potential project, a project of type $\alpha = 1$, generates an expected net present value of $-D + \nu B + 2$. Low values for the overall net present value suggest (though this result clearly depends on the distribution of types) that a high proportion of projects are inefficient. Note that $D \geq 2 + \nu B$ implies that no projects

[15] We model this option to stop the project as a costless liquidation at a very early stage; however, supposing that the agent could recover a sufficiently large salvage value at an early stage would generate similar qualitative results.

should be funded, whereas $\nu B \geq D$ implies that all projects are efficient and should be funded. With intermediate values of the net present value of a project, only projects with $\alpha \geq \underline{\alpha}^* := \frac{D-\nu B}{2}$ are efficient.

The following diagram summarizes both the borrower's actions and the payoffs required and generated by the investment project.



Summary of Timing

Borrowers and lenders are risk-neutral, and every agent seeks to maximize the sum of his first- and second-period incomes.

2.3 Simplifying assumptions

We add several auxiliary assumptions that help to simplify the analysis.

First, we assume that a borrower cannot owe more than she can possibly repay in the best possible state (that is, no more than $B + 2\alpha$). This assumption can be understood as a no-fraud condition that prevents borrowers from consuming in the interim period with the intention of defaulting for sure in the future.[16] This assumption is reasonable, as most legal systems allow for punishment above limited liability (i.e., prison or personal liability) if it is found (perhaps with some probability) that a borrower did not intend to repay in any possible state of the world.[17]

[16] As the hidden sector is more expensive than bank borrowing, no borrower accesses funds from the hidden sector to consume in the interim period and repay in the good state. Therefore, borrowing to consume in the interim period would be worthwhile only if the borrower intended to default for sure.

[17] Note that such a borrowing limit requires the payoff to become verifiable in case of default. We believe that it is plausible that if the project fails, triggering liquidation and investigation, α becomes verifiable, but in the absence of liquidation proceedings, it is not. Introducing a small verification cost in Period 2,

Next, we assume that the initial loan from the banking sector is exactly D and that each borrower uses only one bank at a time. Note that this exclusivity restriction is easy to enforce, given our assumed information structure, with a covenant that deems a contract void if the borrower approaches another bank. Other contracts may arise in the absence of this assumption, particularly contracts in which the lending of one bank is conditional on the lending of others. However, the equilibrium outcome under exclusivity is one in which welfare at $t = 0$ is maximized, conditional only on incentive-compatibility constraints and a break-even condition on banks, so it would continue to be an equilibrium under more-general assumptions. By contrast, the contracts between banks and hidden lenders are, by construction, non-exclusive.

Finally, we impose parametric restrictions that preclude some trivial and uninteresting cases. Specifically, we assume that $D > 2$, which ensures that no borrower can repay for sure, and $D > \nu B > D - 2$, which ensures that each type of borrower will default to a differing extent if the project is unsuccessful (thus, from the lenders' perspective, they are different types). In particular, the second restriction implies that some projects are efficient and should be funded, whereas others are not.

3 Equilibrium

A bank offer at $t = 0$ consists of a contract consisting of repayment schedules $\{(p, q(p))\}$. This offer could depend on the full history of offers up to that point. After the initial stage of bank competition is concluded and the borrower has accepted an offer, she has to decide, at $t = \frac{1}{2}$, whether to pursue the project or to liquidate. If the borrower does not liquidate, she must decide, at $t = 1$, which schedule from the current contract to choose. As described above, the borrower can choose to stay with her current bank for the final period or switch to a simple (p, q) offer from another bank. Note that if on the final schedule chosen, $p > \alpha$, the borrower must fund any shortfall in the first payment by borrowing from the hidden source.

Consider a borrower of type α, who takes a repayment schedule $(p,\, q)$. First, suppose that $\alpha \geq p$ such that there are spare cash flows at $t = 1$ after paying p. Note that it is optimal for the borrower to consume these remaining cash flows rather than keep them as savings for the final period because in the final period, she would only consume them with probability ν. Furthermore, the borrower would not borrow from the hidden

in the spirit of the costly state verification literature (Townsend, 1979; Gale and Hellwig, 1985), would not affect the qualitative results.

lender to consume in the interim period because for each unit of additional consumption in the interim period she would pay, in expectation, $\nu r > 1$ in the final period. Thus, overall, the borrower's expected payoff, if $\alpha \geq p$, is $(\alpha - p) + \nu(B + \alpha - q)$, where the first term corresponds to consumption at $t = 1$ and the second term corresponds to expected consumption at $t = 2$. Similarly, if $\alpha < p$, the borrower's expected overall payoff is $\nu(B + \alpha - q - r(p - \alpha))$. The borrower does not consume in period $t = 1$; she borrows $(p - \alpha)$ from the hidden lender and repays $r(p - \alpha)$ at $t = 2$.

We begin by outlining some preliminary results that place limits on the repayment schedules that the banking sector might offer in equilibrium. The first result is intuitive: loosely, more credit-worthy borrowers are relatively more likely to prefer repayment schedules involving higher interim payments.

Lemma 1 *Consider two borrower types, a and b, and two schedules (p_1, q_1) and (p_2, q_2), where $p_1 > p_2$. If the b-type borrower would choose the former schedule over the latter and the a-type would choose the latter, then $b > a$.*

Proof. The proof of this and all subsequent results appear in the Appendix. ■

This lemma is proved in the Appendix using the incentive constraints of both types of borrowers. A higher-type borrower ($b > a$) can always mimic the choices of a lower-type borrower, with no additional penalties and (at least) a gain in consumption in the interim period of $(b-a)$; a lower-type borrower may instead have to engage in costly hidden borrowing to mimic the behavior of a higher-type borrower. Thus, it cannot be the case that a lower-type gains higher utility by making a relatively high interim payment, as the higher-type borrower can always mimic such behavior.

As a consequence of Lemma 1, it follows that repayment schedules are taken up by intervals of borrowers (that is, no repayment schedule is taken up by a disjoint set of borrower types).

Lemma 2 *Consider borrower types $a < b < c$. If a and c would choose schedule (p_1, q_1) over schedule (p_2, q_2), then b would also choose schedule (p_1, q_1) over (p_2, q_2).*

Corollary 1 *No repayment schedule is taken up in equilibrium by a disjoint set of borrower types; equivalently, every schedule is taken up by an interval of borrower types.*

These results allow us to prove the following, more substantive properties of equilibrium contracts. While this result characterizes properties of equilibria in our model, it does not speak to existence. We address equilibrium existence by construction in Propositions 2 and 4.

Proposition 1 *Any contract schedule* (p, q) *that is adopted in equilibrium has the interim payment* p *set at the* $t = 1$ *cashflow of the highest type adopting the schedule and the final period payment* q *set so that the schedule breaks even. That is, if* (a, b) *is the interval of borrower types that takes up the* (p, q) *schedule, then* $p = b$ *and* $q = \frac{D-b-(1-\nu)E(\alpha|\alpha\in(a,b))}{\nu}$.

Each repayment schedule breaks even; that is, there can be no cross-subsidies between borrowers who are perceived as different by the banking sector or, equivalently, conditional on the information known in the banking sector, every repayment schedule must break even for each observationally distinct type of borrower. This is a result of competition among banks.

Further, we argue that the interim payment from any pool of borrowers (a, b) is equal to the interim cashflow of the highest type; that is, $p = b$. The intuition here is, first, that if $p < b$, it is less costly for the b-type borrower to pay a higher interim payment than it is for a type who would need to borrow from the hidden lender to access the same contract. Thus, a rival bank can cherry-pick these higher types of borrowers at the interim period by offering a slightly higher interim payment and a lower final payment. However, this situation cannot occur in equilibrium, as this would entail losses for the original lending bank. Attempts to cherry-pick by offering an interim payment higher than b would not be successful: the incremental cost for the b-type borrower of hidden borrowing to access such a contract is identical to the incremental cost for all lower types (because the cost of hidden borrowing is linear in the amount borrowed) and so a contract that attracts the b-type borrower attracts all lower types (but involves costly hidden borrowing and thus leads to a less efficient outcome, making it unprofitable).

Formally, the proof proceeds by contradiction. If the proposition does not hold, then a bank has a profitable deviation to offer a contract of this form—that is, one that would attract an interval of borrower types—and sets the interim payment at the interim cashflow of the highest type and the final payment at the break-even point. First, we demonstrate that such a deviation is feasible. Next, we demonstrate that subsequent opportunities for rival offers cannot render such a contract unprofitable.

The proof strategy relies on the potentially unlimited sequence of offers and the structure of competition between banks, described in Section 2.1. The intuition here is that if a bank observes a set of borrowers that are subsidizing other borrowers during a round of competition, it could offer improved terms only to them. These borrowers would switch banks, leaving their previous bank with only subsidized borrowers and thus losses. This is the case in both stages of competition and implies that contracts must break even overall. Similarly, the nature of competition ensures that the intuition and proof of Proposition 1 can be easily extended from the contract offered in equilibrium at $t = 0$ to any deviation offer at $t = 0$.[18]

The absence of cross-subsidies, as a consequence of competition combined with information-sharing, also implies that a given bank cannot replicate the hidden lender. A bank could be tempted to set up a line of credit that resembles the hidden lender and commit not to act upon its own information. However, the borrowing positions in this line of credit would still be visible to other banks. If borrowers used this alternative credit line instead of the hidden lender, the bank would internalize hidden borrowing. Competition at $t = 0$ would imply that these internalized profits would be passed to the borrowers, which would entail cross-subsidies across borrowers that are perfectly observable to other banks. At $t = 1$, rival banks could target those borrowers who are implicitly subsidizing others, leaving the original bank with losses.

Noting that the cost of hidden borrowing is continuous in the amount borrowed and that types are continuously distributed, it is immediate that there must be an indifferent type between any two equilibrium contract schedules. Thus, for two pools of borrowers (a, b) and (b, c), as the first pool makes an interim payment of b and a second-period payment of $\frac{D-b-(1-\nu)E(\alpha|\alpha\in(a,b))}{\nu}$, following Proposition 1, and the second pool makes an interim payment of b and a second-period payment of $\frac{D-c-(1-\nu)E(\alpha|\alpha\in(b,c))}{\nu}$, indifference of a b-type borrower implies that

$$\frac{\nu r - 1}{1 - \nu}(c - b) = E(\alpha|\alpha \in [b, c]) - E(\alpha|\alpha \in (a, b)). \tag{1}$$

Similarly, if there is a lowest pool of borrower types, $(\underline{\alpha}, d)$, then as the lowest borrower

[18] Note that with simultaneous, rather than sequential, bank offers at $t = 1$, the equilibrium might rely on multiple different simple contracts offered and adopted at $t = 1$ among a set of borrowers that take an element of the initial $t = 0$ menu. In this case, a competing bank could not target a particular subset of borrowers who adopt a specific contract offered at $t = 1$, as the simultaneity of offers implies that the set of borrowers who take up any contract is not determined until all offers have been made. In particular, an offer intended to target such a subset of borrowers might also attract other types of borrowers who, in the hypothesized equilibrium would take up different $t = 1$ offers. This observation implies that the proof of Proposition 1 would not apply in the case of simultaneous offers and that the analysis of that case may be involved.

type $\underline{\alpha}$ is either indifferent between borrowing and liquidating or all types borrow, it follows that $\underline{\alpha} = \max\{0, \widetilde{\alpha}\}$, where

$$\frac{\nu r - 1}{1 - \nu}(d - \widetilde{\alpha}) = E(\alpha | \alpha \in (\widetilde{\alpha}, d]) - \frac{D - \nu B}{1 - \nu} + \widetilde{\alpha}\frac{1 + \nu}{1 - \nu}. \tag{2}$$

If the equilibrium implies a finite number of pools, we can characterize them by cutoffs $\alpha_1 > \alpha_2 > ... > \alpha_n = \underline{\alpha}$. Expressions (1) and (2) define a system of n equations in the n unknowns $\alpha_1, ..., \alpha_n$. For given parameter values and distributions of types, it is straightforward to determine the solution (or possibly solutions) for all values of n and to assess the feasibility of these candidate equilibria (that is, ensure that the solutions are in the range $1 > \alpha_1 > ... > \alpha_n \geq 0$). We refer to this equilibrium structure as an n-tranche equilibrium.[19]

Equation (1) is a direct implication of the incentive-compatibility constraint of the lowest member of a pool. This result has intuitive implications. A large pool (a, b) is easier to sustain when r is low—so that imitating better types is inexpensive—and when the average quality of the borrowers in the next-lower pool (b, c) is worse. Given one pool, equation (1) determines the sizes of the pools immediately above and below it, generating $n - 1$ conditions for n pools. Equation (2) uses the indifference of the last borrower of the last pool between investing or liquidating (or the possibility that all types borrow) and is the condition that closes the system.

Among contracts that satisfy these restrictions, the assumption that borrowers do not know their own type at $t = 0$ selects the contract that maximizes ex-ante welfare. The full equilibrium configuration is observed to depend crucially on the interest rate at which the hidden sector lends. In particular, if the interest rate is sufficiently high ($r > \frac{2-\nu}{\nu}$), then, as we argue below, opaque lending is too expensive to be used to conceal a bad realization of α, making it irrelevant. If, instead, the interest rate is sufficiently low, then it is easy for lower-type borrowers to mimic higher-type borrowers.

Regardless of the amount borrowed, the opaque lender will always be repaid if the good state is realized and will always face default in the bad state. This follows from the seniority of bank debt. Thus, the break-even rate for r is $\frac{1}{\nu}$, regardless of the pool of borrowers that is attracted by the hidden lender. This rate would be the endogenous rate for the opaque sector if there were no other frictions or inefficiencies. However, whether we think of the

[19] In fact, in the case that we analyze below, with a uniform distribution of borrower types, we show that the equilibrium features an infinite number of pools.

opaque lending sector as trade credit, a credit card, personal loans to an entrepreneur, or an informal lender, it is reasonable to believe that the interest rate charged could be above this break-even rate—for example, if there were other uses or users of this source of lending. Therefore, we study the situations in which $r > \frac{1}{\nu}$.[20]

3.1 Expensive hidden borrowing

In this section, we explore the implications of an expensive opaque sector. When the interest rate r is larger than $\frac{2-\nu}{\nu}$, borrowing from the opaque lender is so expensive that it is irrelevant. As a result, there is full separation among those types that borrow—that is, each borrowing type repays the banking sector a different interim payment. Thus, in this contract, the implicit interest rates between $t = 1$ and $t = 2$ are fully contingent upon p. The intuition is that there is no opportunity for banks, at $t = 1$, to offer a more attractive contract to any borrower—contingent on observable information, a borrower's surplus is maximized; here, the observable information *is*, in effect, the agent's type. Further, borrowers retain all of the surplus. Consequently, the marginal type $\underline{\alpha}$, indifferent between taking a contract or liquidating, is $\underline{\alpha}^*$; that is, liquidation decisions are efficient. Therefore, overall surplus is maximized and captured by borrowers. This outcome is preferred by borrowers at the *ex-ante* stage, where contracts are determined.

Proposition 2 *When the opaque sector lends at a sufficiently high interest rate ($r > \frac{2-\nu}{\nu}$), there exists a fully separating equilibrium where where the first bank makes the following offer (and no other banks make offers). This offer is a contract where the interim payment is equal to the first-period cashflow and the corresponding final payment fully reflects the information implied by the revealed first-period cashflow. Liquidation at $t = \frac{1}{2}$ is at the efficient level, $\underline{\alpha}^* = \frac{D-\nu B}{2}$ and the equilibrium achieves the first-best outcome.*

The two necessary incentive-compatibility conditions that are fundamental to this contract provide a useful intuition. First, the contract must guarantee that no borrower wants to imitate a lower-quality borrower. This condition is always true under full separation. For a borrower of type α, imitating a lower type a provides additional consumption $(\alpha - a)$

[20] Note that the model implies a break-even rate that is independent of the amount borrowed, and we simply assume that the markup that the hidden lender charges above the break-even rate is also independent of the amount borrowed. In application, this assumption is justified because borrowers may be able to obtain several small loans from different lenders (e.g., different credit cards, different payday lenders, or both simultaneously). Given that opaque lenders do not share information, they cannot condition on other loans.

at $t = 1$ and an increase in the $t = 2$ expected repayment of $(2 - \nu)(\alpha - a)$, which always exceeds the additional consumption, as $\nu < 1$. Second, the contract must be such that no borrower wants to imitate a higher-quality borrower. A borrower of type α who wants to imitate a higher borrower type b must borrow $(b - \alpha)$ from the hidden lender. Such borrowing entails, in the case of success, a further payment in $t = 2$ of $r(b - \alpha)$ to the hidden lender and a decrease in the payment to the bank of $\frac{(2-\nu)}{\nu}(b - \alpha)$. Therefore, the condition holds if and only if $r > \frac{2-\nu}{\nu}$.

In the equilibrium described in Proposition 2, banks breaks even on a type-by-type basis, and borrowers fully internalize the surplus of their projects. Therefore, there are no possible alternative offers to borrowers at $t = 1$ that attract borrowers without entailing losses. Moreover, liquidation is efficient and the contract achieves the first-best outcome, so there is no scope for rival banks to profitably offer any more attractive contract at the initial stage.

Note that if there is no hidden lender, then as a corollary of Proposition 2, the outcome is first-best. This result follows because the absence of a hidden lender corresponds to an interest rate of infinity ($r \to \infty$) in the opaque sector.

Borrowers obtain all of the surplus generated, as banks are competitive and thus earn no profits. Because hidden lenders are prohibitively expensive, they are inactive. Thus, with expensive hidden lending, the first-best solution is achieved, and borrowers retain all of the surplus from projects that are financed.

3.2 Inexpensive hidden borrowing

In this section, we explore equilibrium outcomes when the opaque sector is relatively inexpensive—that is, when $r < \frac{2-\nu}{\nu}$.

First, we demonstrate that there will be some pooling among different types of borrowers with regard to their interim payments. Furthermore, banks cannot distinguish among the different types within a pool of borrowers who all make the same interim repayment, and because banks break even within each observable pool of borrowers, it follows that within each pool, borrowers cross-subsidize each other.

Proposition 3 *When the hidden lender's interest rate is sufficiently low ($r < \frac{2-\nu}{\nu}$), there cannot be an equilibrium in which a continuum of borrowers are able to fully separate.*

The intuition behind this proposition is that if two similar types can fully separate, then by borrowing "a little" from the hidden lender, a lower-type borrower can mimic a higher-type borrower and will be better off overall. That is, by borrowing marginally, the borrower can affect the interest rate on infra-marginal outstanding debt. As a result, Proposition 3 implies that when $r < \frac{2-\nu}{\nu}$, in any equilibrium, all borrowers belong to some pool—i.e., no borrower can fully separate.

Thus, when $r < \frac{2-\nu}{\nu}$, there is some pooling in the choices of different borrower types. In contrast, Proposition 2 demonstrates that when $r > \frac{2-\nu}{\nu}$, there is full separation of borrower types and a continuum of repayment schedules or contract contingencies. It follows that there is a smaller number of contract contingencies when $r < \frac{2-\nu}{\nu}$ or, in other words, contracts are simpler or, equivalently, less flexible when hidden borrowing is relatively inexpensive.

Following Proposition 3, the presence of a relatively inexpensive hidden lender restricts banks' contractual options, making the contract less contingent on intermediate payments, and taking the form characterized in Proposition 1. As the interest rate of the hidden lender falls, banks find it harder to distinguish between borrowers. Note that within a pool of indistinguishable borrowers, the bank interest rate between $t = 1$ and $t = 2$ is the same for all borrowers regardless of their creditworthiness. Within this pool of indistinguishable borrowers, higher-quality borrowers cross-subsidize lower-quality borrowers.

In general, the lower the cost of borrowing from the hidden sector, the more easily a lower type of borrower can imitate a marginally better type and, intuitively, an infra-marginally better type of borrower. This intuition suggests that the top pool of borrowers, those who take the same bank terms as a borrower of type 1, must become broader as the cost of borrowing from the hidden lender decreases. Equivalently, a lower cost of hidden borrowing leads to a greater range of different types of borrowers adopting the same repayment schedule. We provide support for this intuition below. To provide a complete characterization, we assume henceforth that types are uniformly distributed.

Proposition 4 *If $\alpha \sim U[0,1]$, there exists an equilibrium where the first bank makes offers as follows (and no other banks make offers):*

1. *If $r \geq \frac{2-\nu}{\nu}$, there is full separation of borrower types with each type $\alpha > \underline{\alpha}^* = \frac{D-\nu B}{2}$ paying α at the interim stage and $q = \frac{D-(2-\nu)\alpha}{\nu}$ as a final payment.*
2. *If $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$, there are countably-infinite pools for $i = 1, 2, ...$ where all borrowers of type $(\alpha_i, \alpha_{i-1}]$ make an interim payment of α_{i-1} and a final payment of $q =$*

$\frac{D-\alpha_{i-1}-(1-\nu)\frac{(\alpha_i+\alpha_{i-1})}{2}}{\nu}$, *where* $\alpha_0 = 1$ *and*

$$\alpha_i = 1 - \frac{2 - r\nu - \nu}{1-\nu}\left(1 - \frac{D - \nu B}{2}\right)\frac{1 - \left(\frac{\nu+2r\nu-3}{1-\nu}\right)^i}{1 - \frac{\nu+2r\nu-3}{1-\nu}}. \tag{3}$$

3. *If* $\frac{3-\nu}{2\nu} > r$*, there is a single pool of borrowers of type* $\alpha \in (\frac{\nu+2D-2B\nu+2r\nu-3}{\nu+2r\nu+1}, 1]$ *who make an interim payment of* 1 *and a final payment of* $q = \frac{B-4r+\nu+2D-B\nu+2r\nu+2rD-3}{\nu+2r\nu+1}$.

The regime in the range $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$, which contains multiple pools, converges to the other two regimes. Within the range $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$, equation (1) requires that the mass of borrowers accessing the n^{th} contract is a fraction $\frac{\nu+2r\nu-3}{1-\nu} \in (0,1)$ of the mass of borrowers accessing the $n-1^{th}$ contract. As $r \to \frac{2-\nu}{\nu}$, the term $\frac{\nu+2r\nu-3}{1-\nu}$ converges to 1, leading to an equilibrium with infinite small pools that are almost-equally-sized, covering the borrowers in the range $(1, \frac{D-B\nu}{2})$, which resembles the fully separating equilibrium. As r grows, each pool n becomes bigger relative to the next-lower pool $n+1$. When $r \to \frac{3-\nu}{2\nu}$, the term $\frac{\nu+2r\nu-3}{1-\nu}$ converges to zero and the first pool covers almost all of the borrowers. When $r > \frac{3-\nu}{2\nu}$, only one pool can exist; the cross-subsidies between borrowers lead to $\alpha_1 < \frac{D-B\nu}{2}$, so liquidation is inefficient.

In the range $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$, the top repayment schedule (high-quality borrowers) accounts for the largest fraction of overall borrowing, and pools become smaller towards the bottom. There are an infinite number of pools, and the bottom pool can be considered arbitrarily small. This consideration implies no cross-subsidies at the very bottom and thus efficient liquidation decisions—i.e., as $i \to \infty$, $\alpha_i \to \frac{D-B\nu}{2}$. Thus, throughout this range of the cost of hidden borrowing, only efficient projects are undertaken and all inefficient projects are liquidated. The following corollary to Proposition 4 demonstrates that as the cost of hidden borrowing decreases, α_1 decreases, and thus the fraction of borrowers who pool in the top contract schedule offered by the banking sector increases. Indeed, this result indicates that as the cost of hidden borrowing falls, the "highest"repayments schedules account for larger shares of borrowers. In this sense, a high cost of hidden borrowing is associated with a greater proportion of borrowers accessing a greater variety of repayment schedules or, in other words, with increased financial complexity in the formal banking sector.

Given Proposition 4, the following corollaries are easily shown:

Corollary 2 *If* $\alpha \sim U[0,1]$*, and* $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$*, then* $\frac{d\alpha_i}{dr} > 0$ *for all* i*.*

Corollary 3 *If* $\alpha \sim U[0,1]$*, welfare is given by*

$$W := \begin{array}{cc} \left(1 - \frac{D-\nu B}{2}\right)^2 & \text{for } r \geq \frac{2-\nu}{\nu} \\ \frac{1+r}{2}\nu\left(1 - \frac{D-\nu B}{2}\right)^2 & \text{for } \frac{2-\nu}{\nu} \geq r > \frac{3-\nu}{2\nu} \\ 8\nu\frac{r+1}{(\nu+2r\nu+1)^2}\left(1 - \frac{D-\nu B}{2}\right)^2 & \text{for } \frac{3-\nu}{2\nu} \geq r \geq \frac{1}{\nu} \end{array}. \tag{4}$$

Welfare is constant in r *for* $r > \frac{2-\nu}{\nu}$*, increasing in* r *for* $\frac{2-\nu}{\nu} \geq r > \frac{3-\nu}{2\nu}$ *and decreasing in* r *for* $\frac{3-\nu}{2\nu} \geq r \geq \frac{1}{\nu}$*.*

Thus, we characterize welfare and demonstrate that it increases as the cost of hidden borrowing increases in the range where more than one repayment schedule is taken up. Within the fully separating equilibrium, the first-best outcome is achieved and welfare is constant. In the range where multiple pools exist, liquidation is efficient, although pools become smaller, as the value of r increases. The reduction in borrowing dominates the higher cost of borrowing and welfare rises with r. In the single-pool equilibrium, a lower r leads to more borrowing and less efficient liquidation; these two effects are dominated by the lower cost of borrowing, so welfare is decreasing in r.

3.3 Interest rates

When $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$, borrowers have a choice from a contract of countably infinite repayment schedules. Each of these has an initial outstanding balance of D and an interest rate from $t = 0$ to $t = 1$ of one. At $t = 1$, the minimum repayment allowed is $\underline{\alpha}^*$, but higher repayments are acceptable and imply lower interest rates on the outstanding balance between $t = 1$ and $t = 2$. We can denote the net interest rate of this second period by $R(p)$. It is implicitly defined by

$$(D - p)(1 + R(p)) = q(p). \tag{5}$$

Given that $p = \alpha_{i-1}$ is the type of the highest borrower of a pool in equilibrium, the break-even condition for the bank is

$$(D - \alpha_{i-1}) = vq(\alpha_{i-1}) + (1 - v)\frac{\alpha_{i-1} + \alpha_i}{2}. \tag{6}$$

Substituting $R(p)$ from (5), we can write

$$(1 + R(\alpha_{i-1})) = \frac{1}{v} - \frac{(1-v)}{v}\frac{\alpha_{i-1}+\alpha_i}{2}\frac{1}{(D-\alpha_{i-1})}. \tag{7}$$

This result implies that the interest rate on the remaining balance is always below the interest rate that the hidden lender charges, as $r > \frac{1}{\nu}$. Moreover, the interest rate is decreasing in the interim payment α_{i-1}.

Similarly, when r is sufficiently high that different borrower types separate fully, the associated second-period interest rates are given by $(1 + R(\alpha)) = \frac{1}{v} - \frac{(1-v)}{v}\frac{\alpha}{(D-\alpha)}$. Finally, when r is sufficiently low that the only acceptable payment in the interim period is 1, the associated interest rate takes a value between 1 and $\frac{1}{v}$ and increases as the rate of the hidden lender, r, decreases. Thus, these properties hold for all values of r.[21]

It is worth restating the implications of the model for the flexibility of interest rates in repayment schedules.

When $r \geq \frac{2-\nu}{\nu}$ interest rates from $t = 1$ to $t = 2$ are fully contingent on the interim payment p. When $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$ interest rates are responsive to the interim payment p, but $R(p)$ is less responsive to the interim payment that the average borrower repays as the value of r decreases. Finally, when $r < \frac{3-\nu}{2\nu}$ the interest rate from $t = 1$ to $t = 2$ and the interim payment p are fixed. In this regime, failure to pay p indicates that α is extremely low (given that concealing low realizations of α is inexpensive) and leads to liquidation.

Note that within pooling contracts, only a limited set of payments p is observed in equilibrium. This is consistent with the interpretation that, as hidden borrowing becomes less expensive, borrowers are less likely to partially default (or prepay) on a loan by a small amount. Thus, interim payments endogenously move further apart. In practice, we do not observe that loan contracts have discrete advance-repayment possibilities. Similarly, in practice, the amount of a partial default is not restricted contractually. However, we can modify the contract offered by banks such that any payment p is allowed in equilibrium in an intuitive manner that yields the same equilibrium outcomes. In particular, the bank contract may allow borrowers to repay $p \in (\alpha_{i-1}, \alpha_{i-2})$ and reduce the outstanding balance by p—rather than being strictly required to pay just α_{i-1}— while maintaining the interest

[21] Both results are true in general and do not rely on the uniform distribution of types. Instead, both the condition that the interest rate on the remaining balance is less than $\frac{1}{\nu}$ and that this interest rate is decreasing in p apply generally. They rely only on the observation that the average borrower of each tranche has creditworthiness that is increasing in p. This is guaranteed by Proposition 1.

rate at $(1 + R(\alpha_{i-1}))$. That is, if borrowers repay more than is stipulated in a given tranche of the contract but less than is stipulated in the subsequent tranche, they reduce their outstanding balance and continue to pay the same interest rate. Such offers will not be taken in equilibrium.

First, consider a borrower who is currently taking the α_{i-1} contract or a schedule with a lower payment. She would need to borrow the additional funds from the hidden lender to make the interim payment p. Given that $R(\alpha_{i-1}) < r$, she would borrow at high rates to reduce her balance at low rates. Second, consider borrowers who, in equilibrium, take the $p_i = \alpha_{i-2}$ schedule or any higher payment. According to our characterization, borrowers do not want to stop borrowing from the hidden lender and consume since that would entail a higher interim interest rate $R(\alpha_{i-1})$. The alternative of a $p_i \in (\alpha_{i-1}, \alpha_{i-2})$, compared to switching to a lower tranche, involves less consumption in exchange for an interest rate $1 + R(\alpha_{i-1})$ that is paid with probability $\frac{1}{\nu}$. Given that $\frac{(1+R(\alpha_{i-1}))}{\nu} < 1$, this is a worse option than consuming and paying α_{i-1}, which we have shown to be dominated. Consequently, such borrowers would also not want to deviate.

4 Conclusions

We have presented a model of financing of an investment project, with central mechanisms that have wide applicability—in particular, to interactions between different sources of borrowing and their implications for contractual form. Our results highlight a possible reason why long-term debt contracts are inflexible with respect to interim payments: the information that long-term lenders would extract from these interim payments is corrupted by additional borrowing from hidden sources of funding. Our results also suggest an explanation for simultaneous borrowing from different sources, even when there is a clear pecking order among them and borrowing from the less expensive source has not been fully exhausted (for example, firm loans and trade credit, or mortgages and credit-card borrowing, when both trade credit and credit-card borrowing are not costlessly observable by the bank). In our model, borrowers may want to use an expensive hidden source of funds to conceal their lack of creditworthiness. Although it is always less expensive to obtain funds from banks, a borrower may use a hidden lender to affect the interest rate that she receives in the banking sector.

The existing literature has drawn a distinction between informal and formal lending and highlighted the fact that the informal sector may increase credit availability through different information and enforcement technologies. In this paper, we focus on opaque lending and, to the extent that informal lending is opaque, highlight an indirect channel through which informal lending may diminish welfare. In particular, opaque lending may indirectly diminish welfare through its effect on lending in the formal sector. Although this indirect channel may be sufficiently strong to generate a net welfare loss, borrowers who would (ex ante) prefer to commit not to access informal lending have no means of doing so and so might be compelled to access informal lending.

The model makes several empirical predictions. In particular, we observe that changes in the efficiency of alternative lending sources may affect the form and nature of bank lending. The effects of alternative lending sources on contractual forms in the banking sector (where higher costs of hidden borrowing lead to a wider variety of contract forms) suggest that as the informational transparency of the financial sector as a whole improves, consumers may take up a wider range or a more sophisticated set of financial instruments from banks. Finally, we predict that borrowers may simultaneously access both expensive, but hidden, sources of credit from an opaque lending sector and less expensive credit from the formal banking sector.

Most of the empirical predictions of the model relate to the levels of efficiency and informational transparency of alternative lenders. Cross-country comparisons indicate substantial differences in the effective levels of information sharing in different countries (Miller, 2003; Jentzsch, 2007). In some countries, such as France, such restrictions as privacy-protection laws have precluded the creation of credit bureaus. In others, the existence of inexpensively accessed- and centralized public credit registers (that do not cover borrowing sources, such as small credits, credit cards, or consumer credit) has crowded out private credit sources. The model predicts that these differences should affect debt-market contracts. Along the same lines, it is suggestive that the higher degree of innovation in mortgage markets in Anglo-Saxon countries has had no counterpart in continental Europe (as suggested in Green and Wachter, 2005).

References

[1] Allen, Franklin (1985): "Repeated Principal-Agent Relationships with Lending and Borrowing," *Economics Letters*, 17, 27-31.

[2] Arnott, Richard J. and Joseph E. Stiglitz (1991): "Equilibrium in Competitive Insurance Markets with Moral Hazard," NBER Working Paper No. W3588.

[3] Bar-Isaac, Heski and Vicente Cuñat (2012) "Long-Term Debt and Hidden Borrowing" Working Paper EC-12-05, Stern School of Business, N.Y.U.

[4] Barron, John and Michael Staten (2003) "The Value of Comprehensive Credit Reports: Lessons from the US Experience"in Miller, Margaret J. ed. *Credit Reporting Systems and the International Economy*, MIT Press, 273-310.

[5] Bennardo, Alberto, Pagano, Marco and Piccolo, Salvatore (2009), "Multiple-Bank Lending, Creditor Rights and Information Sharing, SSRN - working paper.

[6] Bisin, Alberto and Danilo Guaitoli (2004): "Moral Hazard and Nonexclusive Contracts Production: Theories and Evidence from Joint Ventures" *RAND Journal of Economics,* 35, 306-328.

[7] Bisin, Alberto and Adriano A. Rampini (2006) "Exclusive Contracts and the Institution of Bankruptcy," *Economic Theory*, 27, 277-304.

[8] Bizer, David S. and Peter M. DeMarzo (1992): "Sequential Banking," *Journal of Political Economy*, 100, 41-61.

[9] Brown, Martin, Jappelli, Tulio, and Marco Pagano (2009),"Information sharing and credit market performance: firm-level evidence from transition countries," *Journal of Financial Intermediation,* 18(2), 151-172.

[10] Cole, Harold L. and Narayana R. Kocherlakota (2001): "Efficient Allocations with Hidden Income and Hidden Storage," *Review of Economic Studies*, 68, 523-542.

[11] Cowen, Kevin and Jose De Gregorio (2003): "Credit Information and Market Performance: The Case of Chile," Chapter 4 in Miller, Margaret J. ed. (2003) *Credit Reporting Systems and the International Economy*, MIT Press, 273-310.

[12] Cornaggia, Kimberly J., Laurel A. Franzen, and Timothy T. Simin (2012) "Managing the Balance Sheet with Operating Leases" SSRN - working paper.

[13] Cornaggia, Kimberly J., Laurel A. Franzen, and Timothy T. Simin, (2013) "Bringing Leased Assets onto the Balance Sheet", Journal of Corporate Finance, fothcoming. 2013

[14] Dionne, Georges and Pierre Lasserre (1985): "Adverse Selection, Repeated Insurance Contracts and Announcement Strategy," *Review of Economic Studies,* 52, 719-723.

[15] Doepke, Matthias and Townsend, Robert M., (2006) "Dynamic mechanism design with hidden income and hidden actions," *Journal of Economic Theory*, Elsevier, vol. 126(1), 235-285.

[16] Elliehausen, G. and E. C. Lawrence, (2001). "Payday advance credit in America: An analysis of consumer demand (Monograph No. 35)". Washington, D.C.: Georgetown University, McDonough School of Business, Credit Research Center.

[17] Gale, Douglas and Martin Hellwig (1985) "Incentive-Compatible Debt Contracts: The

One-Period Problem," *Review of Economic Studies*, 647-663.

[18] Green, Richard K. and Susan M. Wachter (2005) "The American Mortgage in Historical and International Context," *Journal of Economic Perspectives,* 19(4), 93-114.

[19] Hunt, Robert M. (2006) "The Development and Regulation of Consumer Credit Reporting in America," in Bertola, Giuseppe and Charles Grant, eds., *The Economics of Consumer Credit: European Experience and Lessons from the US*, Boston: MIT Press.

[20] Innes, Robert (1990) "Limited Liability and Incentive Contracting With Ex-Ante Action Choices," *Journal of Economic Theory*, 52, 45-68.

[21] Jaynes, Gerald D. (1979) "Equilibria in Monopolistically Competitive Insurance Markets," *Journal of Economic Theory*, 19, 394-422.

[22] Jappelli, Tulio and Marco Pagano (1993) "Information Sharing in Credit Markets," *Journal of Finance*, 43(5), 1693-1718.

[23] Jappelli, Tulio, and Marco Pagano (2006) "Role and Effects of Credit Information Sharing," in *The Economics of Consumer Credit*, edited by G. Bertola, R. Disney and C. Grant. Cambridge: MIT Press, 347-371.

[24] Jensen, Michael C. and William H. Meckling (1976) "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics* Volume 3, Issue 4, October, 305–360.

[25] Jentzsch, Nicola (2007) *Financial Privacy - An International Comparison of Credit Reporting Systems* (*Springer-Verlag, Heidelberg*), $2^{\text{nd.}}$ edition.

[26] Ljungqvist, Lars and Thomas Sargent (2004) *Recursive Macroeconomic Theory,* MIT Press.

[27] Mann R. J. and J. Hawkins (2007) "Just Until Payday," *UCLA Law Review,* 54 – 4.

[28] Miller, Margaret J. ed. (2003) *Credit Reporting Systems and the International Economy*, MIT Press, 273-310.

[29] Morse, Adair (2011) "Payday Lenders: Heroes or Villains?" Journal of Financial Economics 102(1) 28-44

[30] Padilla, A. J., and M. Pagano. "Endogenous Communication Among Lenders and Entrepreneurial Incentives," *Review of Financial Studies,* 10(1), 205-23.

[31] Sundareesan, Suresh M. (2009) *Fixed income Markets and their Derivatives*, Elsevier, Oxford, UK.

[32] Townsend, Robert M. (1979) "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory*, 21(2), 265-293.

[33] Zechman, Sarah L. C. (2010), "The relation between voluntary disclosure and financial reporting: Evidence from synthetic leases" *Journal of Accounting Research.* 48(3), 725-765 June.

A Omitted Proofs

Proof of Lemma 1

Proof. Suppose, for contradiction, that this is false. Then, there is some $a \geq b$ such that the a-type chooses the (p_2, q_2) schedule and the b-type chooses the (p_1, q_1) with $p_1 > p_2$.

It cannot be the case that $a = b$ since preference and the tie-breaking rule in case of indifference would ensure that this type would choose only one of the two contracts. Thus $a > b$ and the following possibilities are exhaustive: (i) $a > b \geq p_1 > p_2$; (ii) $p_1 > p_2 \geq a > b$; (iii) $p_1 \geq a > b \geq p_2$; (iv) $p_1 \geq a > p_2 \geq b$; (v) $a > p_1 > p_2 > b$; and (vi) $a > p_1 > b > p_2$.

In case (i) the preference condition for both a and b are identical (and require $vq_2 - \nu q_1 \geq p_2 - p_1$). In case of strict preference, it is immediate that a and b, cannot take up different contracts. In case of indifference, the tie-breaking rule selects the same contract for both types, again providing a contradiction. Similarly in case (ii), the conditions for a borrower of type b to prefer repayment schedule (p_1, q_1) and (p_2, q_2) are identical to the conditions for a borrower of type a to have the same preferences. Thus, it cannot be the case that the b-type strictly prefers one contract and the a-type strictly prefers the other; again, in case of indifference, the tie-breaking rule would imply that both would prefer the same schedule, contradicting that they prefer different schedules.

In case (iii), a takes up the (p_2, q_2) schedule when

$$a - p_2 + \nu(B + a - q_2) \geq \nu(B + a - q_1 - r(p_1 - a)), \tag{8}$$

if and only if

$$(\nu r p_1 - p_2) + \nu(q_1 - q_2) \geq (\nu r - 1)a. \tag{9}$$

Similarly, b prefers the (p_1, q_1) schedule when

$$(\nu r p_1 - p_2) + \nu(q_1 - q_2) \leq (\nu r - 1)b. \tag{10}$$

Putting (9) and (10) together, we obtain $(\nu r - 1)b \geq (\nu r - 1)a$. As $\nu r > 1$ and $a > b$, this result is a contradiction.

In case (iv), similarly to case (iii), a takes up the (p_2, q_2) schedule when

$$(\nu r p_1 - p_2) + \nu(q_1 - q_2) \geq (\nu r - 1)a, \tag{11}$$

and b takes up the (p_1, q_1) schedule when

$$\nu(B + b - q_1 - r(p_1 - b)) \geq \nu(B + b - q_2 - r(p_2 - b)), \tag{12}$$

if and only if

$$q_2 - q_1 \geq r(p_1 - p_2). \tag{13}$$

Combining the two constraints, we have $p_2(\nu r - 1) \geq a(\nu r - 1)$, which contradicts $a > p_2$, a premise of this case, given that $\nu r > 1$.

Next, consider case (v). Then, a takes up the (p_2, q_2) schedule when

$$a - p_2 + \nu(B + a - q_2) \geq a - p_1 + \nu(B + a - q_1), \tag{14}$$

or, equivalently,

$$p_1 - p_2 \geq \nu(q_2 - q_1), \tag{15}$$

and b prefers the (p_1, q_1) schedule when (13) holds. Combining these two conditions, we have $\frac{1}{\nu}(p_1 - p_2) \geq q_2 - q_1 \geq r(p_1 - p_2)$ or $1 \geq r\nu$, also a contradiction.

Finally, consider case (vi). In this case, borrower a prefers the (p_2, q_2) schedule when (15) holds, and b prefers the (p_1, q_1) schedule when (10) holds. Combining these two conditions yields $p_1 - p_2 \geq (\nu r p_1 - p_2) - (\nu r - 1)b$ or $(\nu r - 1)b \geq (\nu r - 1)p_1$, which is false, as $r\nu > 1$, and by assumption, in this case, $p_1 > b$.

This exhausts all possibilities. ■

Proof of Lemma 2

Proof. Suppose, for contradiction, that b takes up (p_2, q_2) over (p_1, q_1). If $p_2 > p_1$, then, following Lemma 1, $b > c$, in contradiction to the premise. If $p_1 = p_2$, then if c strictly prefers the (p_1, q_1) schedule, it must be the case that $q_1 < q_2$ and thus, b would strictly prefer the (p_1, q_1) schedule, providing a contradiction. If $p_2 > p_1$, then following Lemma 1, as a takes up (p_1, q_1), $a > b$, which also contradicts the premise that $c > b > a$. This exhausts all possibilities. ■

Proof of Corollary 1

Proof. As types are continuously distributed, if a contract (p_1, q_1) were taken up by a disjoint set of borrowers, then it would be possible to find $a < b < c$ where a and c take up the (p_1, q_1) schedule and either b takes up no contract or a different schedule (p_2, q_2). The latter case contradicts Lemma 2. The former can be ruled out by noting that the a-type borrower expects a payoff of $(a - p_1)1_{a \geq p_1} + \nu(B + a - q_1 - r(p_1 - a)1_{p > a}) \geq 0$ by taking the (p_1, q_1) schedule, where $1_{a \geq p_1}$ is an indicator that take a value of 1 when $a \geq p_1$. The b-type expects a payoff of $(b - p_1)1_{b \geq p_1} + \nu(B + b - q_1 - r(p_1 - b)1_{p_1 > b}) > (a - p_1)1_{a \geq p_1} + \nu(B + a - q_1 - r(p_1 - a)1_{p_1 > a}) \geq 0$ by taking the (p_1, q_1) schedule, where the first inequality follows from $b > a$. Thus, the b-type borrower strictly prefers taking up the (p_1, q_1) schedule to taking up no contract. ■

Proof of Proposition 1

Proof. Suppose, for contradiction, that the statement of the proposition is false. We proceed by considering a deviant offer (p', q') and show that this deviant offer does not make losses, attracts borrowers, and will not be rendered unprofitable by subsequent (feasible) offers. Given our assumptions on bank preferences, such an offer would, therefore, arise and thereby provide a contradiction.

We begin by noting that an equilibrium cannot involve a bank making losses overall. Thus, the statement of the proposition is false if either (i) there is an equilibrium that includes at least one schedule (p, q) that is strictly profitable, or (ii) all schedules break even, but there is some interval of borrower types, (a, b), that takes up a (p, q) schedule with $p \neq b$.

More formally, given Corollary 1, case (i) states that there exists an interval of borrowers, (a, b), that comprises those borrowers who, in equilibrium, adopt the same repayment schedule (p, q), such that this schedule is strictly profitable for the bank; that is,

$$p + \nu q + (1 - \nu)E(\alpha | \alpha \in (a, b)) - D > 0. \tag{16}$$

For both cases (i) and (ii), consider an offer (p', q') by a rival bank at $t = 1$ to the set of borrowers (a, b) who take up the (p, q) contract. Specifically, set $p' = b$ and q' at the break-even rate given the set of borrowers who prefer the (b, q') contract to the (p, q) contract; that is, the subset $A' \subseteq (a, b)$ consisting of all $\alpha \in (a, b)$ that prefer $(b,\, q')$ to $(p,\, q)$. In particular, as bank profits are given by $b + \nu q' + (1 - \nu)E(\alpha|\alpha \in A') - D$, it follows that $q' = \frac{D-b-(1-\nu)E(\alpha|\alpha\in A')}{\nu}$. Lemma 3, below, establishes that there is at least one such A' that is an upper sub-interval of the original set. Note that there may be more than one (A', q') consistent with this definition, in which case we can pick any of them.

It is convenient to introduce the following notation $q'(c) = \frac{D-b-(1-\nu)E(\alpha|\alpha\in(c,b))}{\nu}$. This corresponds to the break even second period payment associated with an interval of borrowers (c, b) and a first period payment $p' = b$.

While Lemma 3 establishes that, in the absence of further offers, a rival bank can attract some borrower without incurring losses by using such an offer (with $p' = b$ and $q' = q'(c)$) in both cases (i) (profitable contract) and (ii) ($p \neq b$), it still remains to show that there are no further feasible offers in subsequent rounds of competition that would leave the p', q' contract with losses or with no borrowers taking it up. We do so in the resumption of the proof of Proposition 1 following the proof of Lemma 3.

Given our assumptions on bank preferences (that rival banks prefer making offers that attract some borrowers), the first rival bank would make the (p', q') offer providing the contradiction and thereby establishing that the proposition is holds. ■

Lemma 3 *Consider a schedule (p, q) and interval of borrowers (a, b), such that if all borrowers (a, b) adopt the (p, q) schedule, either it earns strictly positive profits or it breaks even and $p \neq b$. There is at least one c with $c \in [a, b)$ such that all $\alpha \in (c, b]$ would choose the contract $(b, q'(c))$ over (p, q).*

Proof. If $p = b$, then the (p, q) schedule must earn strictly positive profits. Therefore $q > \frac{D-b-(1-\nu)E(\alpha|\alpha\in(a,b))}{\nu} = q'(a)$. Trivially, all $\alpha \in (a, b)$ strictly prefer the contract $(b, q'(a))$ to (b, q).

Next, if $p \neq b$ we demonstrate that for all $c \in (a, b)$, b prefers a contract with $p' = b$ and $q' = q'(c)$ to the original (p, q) schedule.

Consider the incentive conditions for b:

i) Suppose that $p < b$. Then, for contradiction, suppose that b weakly prefers (p, q) to (b, q'), or equivalently $b - p + \nu(B + b - q) \geq \nu(B + b - q')$, which implies

$$q' \geq q - \frac{b - p}{\nu}. \tag{17}$$

The original contract earns weakly positive profits which implies that

$$q \geq \frac{D - p - (1 - \nu)E(\alpha|\alpha \in (a, b))}{\nu}. \tag{18}$$

By definition

$$q'(c) = \frac{D - b - (1 - \nu)E(\alpha|\alpha \in (c, b))}{\nu}. \tag{19}$$

These three expressions together imply that

$$\frac{D-b-(1-\nu)E(\alpha|\alpha\in(c,b))}{\nu}\geq\frac{D-p-(1-\nu)E(\alpha|\alpha\in(a,b))}{\nu}-\frac{b-p}{\nu}, \tag{20}$$

or, equivalently, $E(\alpha|\alpha\in(c,b))\leq E(\alpha|\alpha\in(a,b))$, which is a contradiction since $c>a$.

ii) Suppose that $p>b$. Then, for contradiction suppose that b weakly prefers (p,q) to (b,q') i.e.:

$$\nu(B+b-q-r(p-b))\geq\nu(B+b-q'), \tag{21}$$

or, equivalently,

$$q'-r(p-b)\geq q. \tag{22}$$

Since $q'=\frac{D-b-(1-\nu)E(\alpha|\alpha\in(c,b))}{\nu}$ and noting that $q\geq\frac{D-p-(1-\nu)E(\alpha|\alpha\in(a,b))}{\nu}$, we obtain

$$\frac{D-b-(1-\nu)E(\alpha|\alpha\in(c,b))}{\nu}-r(p-b)\geq\frac{D-p-(1-\nu)E(\alpha|\alpha\in(a,b))}{\nu}. \tag{23}$$

Rearranging this inequality implies that

$$E(\alpha|\alpha\in(a,b))-\frac{\nu r-1}{1-\nu}(p-b)\geq E(\alpha|\alpha\in(c,b)). \tag{24}$$

Noting that $E(\alpha|\alpha\in(a,b))\leq E(\alpha|\alpha\in(c,b))$, $\nu r-1>0$, and $p>b$, we find a contradiction.

Thus, in i) and ii) we have shown that b prefers the contract $p'=b$ and $q'=q'(c)$ for any $c\in(a,b)$. It remains to show that there is a $c\in[a,b)$ such that all borrower-types above c would choose the contract $(b,q'(c))$ over the existing (p,q) schedule, and all borrower-types below c would choose the (p,q) schedule over $(b,q'(c))$.

If a prefers the contract $p'=b$ and $q'=q'(a)$ the lemma is satisfied. This is trivially so in case (ii) since the preference condition is the same for all borrower types when contracts entail hidden borrowing, and it is true following Lemma 2 in case (i). Otherwise, note that a lower value of c implies a higher value of $q'(c)=\frac{D-b-(1-\nu)E(\alpha|\alpha\in(c,b))}{\nu}$. Consequently, since b strictly prefers any contract of this form, by continuity there is some indifferent $c\in(a,b)$ and by Lemma 1, all borrowers in $(c,b]$ would choose the contract $p'=b$ and $q'=q'(c)$ over the original (p,q) schedule and all borrowers in $[a,c)$ would choose the original schedule. ■

Resumption of proof of Proposition 1

Proof. We have shown in Lemma 3 that if a (p,q) schedule is not of the form $p=b$ and $q=\frac{D-b-(1-\nu)E(\alpha|\alpha\in(a,b))}{\nu}$, then there is always an alternative contract that, when offered to (a,b), attracts some borrowers. Trivially, the kind of offer described in Lemma 3 which sets $p'=b$ and $q'=\frac{D-b-(1-\nu)E(\alpha|\alpha\in(c,b))}{\nu}$ for some $c\in[a,b)$ attracts borrowers and does not make losses in the absence of further offers.

Next, to demonstrate that a rival bank will offer the (b,q') contract, we must prove that it will not become loss-making and will retain borrowers in subsequent rounds of (weakly profitable) offers that retain some borrowers.

We consider subsequent offers to the set of borrowers (c,b) who adopt the (b,q') contract and demonstrate that they either entail losses, so that no such offers are made, or leave the (b,q') contract with weakly profitable borrowers.

Specifically, suppose that a rival bank in the interim period offers a contract $(\widetilde{p},\widetilde{q})$.We consider cases i) $\widetilde{p}=b$, ii) $\widetilde{p}>b$ and iii) $\widetilde{p}<b$ separately.

i) If $\widetilde{p}=b$ then if $\widetilde{q}\geq q'$, then no borrower would take the $(\widetilde{p},\widetilde{q})$ contract. Instead, if $\widetilde{q}<q'$, then all borrowers in (a,c) would take the $(\widetilde{p},\widetilde{q})$. Therefore, by definition of q', the $(\widetilde{p},\widetilde{q})$ offer would entail losses.

ii) Suppose that $\widetilde{p}>b$. The incentive condition for a generic borrower α in (c,b) to prefer the $(\widetilde{p},\widetilde{q})$ contract to the (b,q') contract is

$$\nu(B+x-q'-r(b-\alpha))<\nu(B+x-\widetilde{q}-r(\widetilde{p}-\alpha)),$$

or, equivalently,

$$\widetilde{q}<q'-r(\widetilde{p}-b),$$

which will either hold for all α in (c,b) or for none.

Note that $r>\frac{1}{\nu}$, so that if $\widetilde{q}<q'-r(\widetilde{p}-b)$, then $\widetilde{q}<q'-\frac{1}{\nu}(\widetilde{p}-b)$. Substituting for q', we have $\widetilde{q}<\frac{D-\widetilde{p}-(1-\nu)E(\alpha|\alpha\in(c,b))}{\nu}$, which means that if the $(\widetilde{p},\widetilde{q})$ contract attracts any borrower, it attracts all of them and entails losses.

iii) Next, suppose that $\widetilde{p}<b$. Then following Lemma 1, the $(\widetilde{p},\widetilde{q})$ contract either attracts all borrowers, or attracts all borrowers in (a,c) while those in (c,b) would choose to remain with the (b,q') contract. The latter case, trivially leaves the (b,q') contract earning strictly positive profits.

Suppose instead that the $(\widetilde{p},\widetilde{q})$ attracts all borrowers in (a,b) and is weakly profitable. As the new contract attracts all borrowers, it attracts the b-type borrower; thus,

$$(b-\widetilde{p})+\nu(B+b-\widetilde{q})>\nu(B+b-q').$$

Equivalently,

$$\frac{(b-\widetilde{p})}{\nu}+q'>\widetilde{q}.$$

For the deviation to be weakly profitable,

$$\widetilde{q}\geq\frac{D-\widetilde{p}-(1-\nu)E(\alpha|\alpha\in A')}{\nu}.$$

Substituting for q', we can combine both inequalities above to obtain:

$$\frac{(b-\widetilde{p})}{\nu}+\frac{D-b-(1-\nu)E(\alpha|\alpha\in A')}{\nu}>\widetilde{q}\geq\frac{D-\widetilde{p}-(1-\nu)E(\alpha|\alpha\in A')}{\nu},$$

which is never true, given that both sides of the inequality are equal, creating a contradiction. Thus, there is no weakly profitable deviation that attracts all borrowers in the pool. ■

Proof of Proposition 2

Proof. To characterize the equilibrium, we can draw on the revelation principle at $t=1$ and consider the borrower's choice from the contract $\{p,q(p)\}$ as a function of her type—that is, we could think of the contract as $\{p(\alpha),q(\alpha)\}$.

Formally, the proposition claims that $p(\alpha) = \alpha$, $q(\alpha) = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$, and that all types $\alpha < \underline{\alpha}^* = \frac{D-\nu B}{2}$ liquidate at $t = \frac{1}{2}$. Efficient liquidation follows, as the marginal type that liquidates is indifferent between liquidating and receiving 0 or continuing the project and receiving an expected a payoff of $(\alpha - p(\alpha))1_{\alpha > p(\alpha)} + \nu(B + \alpha - q(\alpha) - r(p(\alpha) - \alpha)1_{\alpha < p(\alpha)})$. Substituting for $p(\alpha)$ and $q(\alpha)$ and equating the expected payoff to zero implies that the marginal borrower is $\underline{\alpha}^*$.

Turning to the characterization of $p(\alpha)$ and $q(\alpha)$: As discussed above, Proposition 1 ensures that under full separation any meaningful schedule—that is, any schedule that is ever taken up in equilibrium—will have $p = \alpha$ and $q = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$.

Furthermore, incentive-compatibility must be satisfied; that is, a borrower of type α must prefer to make a first-period payment $p(\alpha)$ to any other p on offer. We analyze the incentive-compatibility condition by considering two deviations: imitating a lower type and imitating a higher type.

Incentive-compatibility condition 1: The contract must guarantee that no borrower wants to imitate a lower-quality borrower. Suppose that a borrower of quality α claims to be a lower-quality borrower $a < \alpha$ by making a first payment $p = a$; in that case, her total utility is

$$(\alpha - a) + \nu(B - \frac{D - a - (1-\nu)a}{\nu} + \alpha). \tag{25}$$

Note that $(\alpha - a)$ is the additional consumption at $t = 1$ that results from reporting a lower type, whereas $(B - \frac{D-a-(1-\nu)a}{\nu} + \alpha)$ is the net consumption in the good state (which occurs with probability ν) after repaying $q(a)$. Instead, by revealing her own type, she would obtain $\nu(B - \frac{D-\alpha-(1-\nu)\alpha}{\nu} + \alpha)$. The difference between this and (25) is

$$-(1-\nu)(\alpha - a) < 0. \tag{26}$$

Thus, it cannot be optimal to claim to be a borrower of a lower type.

Incentive-compatibility condition 2: The contract also must guarantee that no borrower wants to imitate a higher-quality borrower by borrowing from the hidden source and making a first payment $p > \alpha$. Suppose, for contradiction, that a borrower claims to be a higher-quality borrower by making a first payment $p = b > \alpha$ and borrowing $b - \alpha$ from the hidden source to fund this payment. The total utility of the borrower would be $\nu(B - \frac{D-b-(1-\nu)b}{\nu} - r(b - \alpha) + \alpha)$ instead of $\nu(B - \frac{D-\alpha-(1-\nu)\alpha}{\nu} + \alpha)$. The difference between the two is

$$(2 - \nu - \nu r)(b - \alpha), \tag{27}$$

which is negative if and only if $r > \frac{2-\nu}{\nu}$. Thus, this is the necessary and sufficient condition for this incentive-compatibility condition to hold.

Finally, in the first-best outcome, a borrower should be funded if and only if the expected NPV of the project is positive—that is, if and only if

$$-D + \alpha + v(B + \alpha) + (1-\nu)\alpha \geq 0. \tag{28}$$

In the candidate equilibrium described above, banks perfectly sort borrowers and offer break-even deals, so borrowers fully internalize the proceeds of their projects. Therefore, the marginal borrower is precisely the one with NPV=0, that is, at the efficient level, and there is no expensive, inefficient borrowing from hidden lenders, so overall efficiency is maximized.

As the contract breaks even on a type-by-type basis, there is no scope for rival banks to make profitable offers at the interim stage. In addition, given that there is efficient liquidation and that the bank breaks even, there is no scope for rival banks to offer any contracts at the initial stage that attract borrowers without incurring losses. Indeed, the existence of this set of repayment schedules as a feasible contract offer suggests that any equilibrium must maximize surplus at the initial stage—in particular, this must involve efficient liquidation and no (inefficient) borrowing from the hidden sector. As a result, the candidate equilibrium is indeed an equilibrium.

It is immediate that the equilibrium involves a borrower of type α paying $p(\alpha) = \alpha$ to the banking sector in the interim period and $q(\alpha) = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$, and all types $\alpha < \frac{D-\nu B}{2}$ liquidating at $t = \frac{1}{2}$. ■

Proof of Proposition 3

Proof. If there were an equilibrium in which borrowers were able to separate, then, following Proposition 1, it would involve $p(\alpha) = \alpha$ and $q(\alpha) = \frac{D-p(\alpha)-(1-\nu)\alpha}{\nu}$ for every borrower type α that borrowed. However, in these circumstances, as suggested by (27), a lower borrower type α would seek to imitate a higher-quality borrower b by borrowing $b - \alpha$ from the hidden borrower. ■

Proof of Proposition 4

Proof. Part 1 follows immediately from Proposition 2.

Next, suppose that $r < \frac{2-\nu}{\nu}$. Consider any two adjacent pools of borrowers (a, b) and (b, c), with $c > b > a$. Then, expression (1) applies, and thus, $\frac{\nu r-1}{1-\nu}(c-b) = E(\alpha|\alpha \in [b, c]) - E(\alpha|\alpha \in [a, b])$. As types are uniformly distributed, this condition can be rewritten as $\frac{\nu r-1}{1-\nu}(c-b) = \frac{c+b}{2} - \frac{a+b}{2}$, or, equivalently, $\frac{b-a}{c-b} = \frac{\nu+2r\nu-3}{1-\nu}$. Note that $\frac{b-a}{c-b} < 1$ if and only if $\gamma := \frac{\nu+2r\nu-3}{1-\nu} < 1$, which is necessarily the case, given that $r < \frac{2-\nu}{\nu}$. Thus, the ratio of the size of any two adjacent pools of borrowers is fixed, and, in particular, a pool of lower-type borrowers is smaller than the adjacent pool of higher-type borrowers. An alternative and convenient way to represent the relative size of two adjacent pools is to compare the size of the higher-type pool to the overall size of both pools (i.e. $\frac{c-b}{c-a}$). This ratio is $\rho := \frac{1-\nu}{2(\nu r-1)} > \frac{1}{2}$. It must be the case that $\rho < 1$ (because $c > b > a$) or, equivalently, $\frac{3-\nu}{2\nu} < r$.

Part 3 is then immediate established. Following the reasoning of the above paragraph, if $r < \frac{3-\nu}{2\nu}$, then, assuming two pools of borrowers with $c > b > a$, we arrive at a contradiction (as this would imply $\rho > 1$, which is inconsistent with $c > b > a$). Consequently, if $r < \frac{3-\nu}{2\nu}$, there can be at most one pool of borrowers and (2) determines the lowest type borrower in this pool as $\underline{\alpha} = \frac{\nu+2D-2B\nu+2r\nu-3}{\nu+2r\nu+1}$. It is easy to verify that $\frac{D-\nu B}{2} > \frac{\nu+2D-2B\nu+2r\nu-3}{\nu+2r\nu+1} > 0$ in this range of r; equivalently, some inefficient projects are not liquidated. It is also easy to calculate the break-even value of q as $\frac{B-4r+\nu+2D-B\nu+2r\nu+2rD-3}{\nu+2r\nu+1}$. This is the only feasible solution, establishing Part 3.

Finally, we turn to Part 2.

First, note that if a given borrower α is accessing a bank loan in equilibrium, then all higher-quality borrower types must also be accessing a bank loan. Otherwise, there would be a contradiction: higher types can mimic the decisions of a lower type and gain non-negative NPV by doing so. In particular, this result implies that if any borrowers are accessing bank loans, then borrowers of type $\alpha = 1$ must also be accessing bank loans.

Consider the pool of borrowers that are choosing the same repayment schedule as the borrower $\alpha = 1$ and suppose that it has mass m. Then, each of the other pools of borrowers must have mass less than m. This result follows from the fact that the ratio between lower- and higher-type pools is $\gamma < 1$. It now follows that if highest type pool has no mass, then all other pools also have no mass. This result is tantamount to full separation and is thus inconsistent with Proposition 3.

Similarly, given that the ratio between lower- and higher-type pools is $\gamma < 1$, it follows that all or almost all (or all but an infinitesimal measure of) borrowers who access bank lending choose from a finite or countable set of schedules.

We denote the marginal borrowers in each pool in equilibrium by $\alpha_0 = 1 > \alpha_1 > \alpha_2 > ... > \alpha_i > ...$ that is, all types $\alpha \in [\alpha_i, \alpha_{i+1})$ choose the same schedule, with $p = \alpha_i$ and q set at the break-even rate, following Proposition 1.

We proceed by (i) characterizing a proposed "n-tranche" candidate equilibrium, where n repayment schedules are offered, (ii) indicating that welfare is increasing in n, so that competition in the first stage leads to a candidate with $n \to \infty$, (iii) demonstrating that the analysis in the limit corresponds to the expressions in the statement of the proposition, (iv) demonstrating that the equilibrium exists, and (v) demonstrating that the equilibrium is essentially unique.

(i) Characterizing an n-tranche equilibrium for uniformly distributed types

We introduce some additional notation and denote the i^{th} threshold borrower in a contract with n tranches or pools of borrowers by $\alpha_i^{(n)}$, where there is ambiguity regarding the number of tranches considered.

An n-tranche equilibrium must satisfy (1), which can be written as $\frac{\alpha_i^{(n)} - \alpha_{i+1}^{(n)}}{\alpha_i^{(n)} - \alpha_{i+2}^{(n)}} = \frac{1-\nu}{2(\nu r - 1)} = \rho$ for $i = 0, 1, ..., n-2$, and (2), which can be written as $\frac{\nu r - 1}{1-\nu}(\alpha_{n-1}^{(n)} - \widetilde{\alpha}_n^{(n)}) = \frac{\widetilde{\alpha}_n^{(n)} + \alpha_{n-1}^{(n)}}{2} - \frac{D - \nu B}{1-\nu} + \widetilde{\alpha}_n^{(n)} \frac{1+\nu}{1-\nu}$, where the lowest borrower type is $\alpha_n^{(n)} = \max\{0, \widetilde{\alpha}_n^{(n)}\}$.

Note that $\alpha_i^{(n)} - \alpha_{i+2}^{(n)} = \alpha_i^{(n)} - \alpha_{i+1}^{(n)} + \alpha_{i+1}^{(n)} - \alpha_{i+2}^{(n)}$, so (1) can be written as

$$\alpha_i^{(n)} - \alpha_{i+1}^{(n)} = \frac{1-\rho}{\rho}(\alpha_{i-1}^{(n)} - \alpha_i^{(n)}) = (\frac{1-\rho}{\rho})^i (1 - \alpha_1^{(n)}) = \gamma^i (1 - \alpha_1^{(n)}). \tag{29}$$

Thus, we can determine the size of any pool of borrowers as a function of its rank, and the size of the top pool.

In addition, for entire mass of borrowers, we can write

$$\begin{aligned}
1 - \alpha_n^{(n)} &= 1 - \alpha_1^{(n)} + (\alpha_1^{(n)} - \alpha_2^{(n)}) + ... + (\alpha_{n-1}^{(n)} - \alpha_n^{(n)}) \\
&= 1 - \alpha_1^{(n)} + \gamma(1 - \alpha_1^{(n)}) + ... + \gamma^{n-1}(1 - \alpha_1^{(n)}) \\
&= (1 - \alpha_1^{(n)}) \frac{1 - \gamma^n}{1 - \gamma}.
\end{aligned} \tag{30}$$

The remaining condition defining the solution to the equations characterizing an n-tranche equilibrium is that either all borrowers prefer to undertake the project than to liquidate or there is a borrower that is indifferent between liquidating or continuing. That is, either $\alpha_n^{(n)} = 0$, with $B + \alpha_n^{(n)} - q_n^{(n)} - r(\alpha_{n-1}^{(n)} - \alpha_n^{(n)}) > 0$, or $\alpha_n^{(n)} > 0$ which requires $B + \alpha_n^{(n)} - q_n^{(n)} - r(\alpha_{n-1}^{(n)} - \alpha_n^{(n)}) = 0$.

We first prove, by contradiction, that $\alpha_n^{(n)} > 0$. Suppose that $\alpha_n^{(n)} = 0$. Then, $B + \alpha_n^{(n)} - \frac{D - \alpha_{n-1}^{(n)} - (1-\nu)\frac{\alpha_n^{(n)} + \alpha_{n-1}^{(n)}}{2}}{\nu} - r(\alpha_{n-1}^{(n)} - \alpha_n^{(n)}) > 0$, so that $B - \frac{D - \alpha_{n-1}^{(n)} - (1-\nu)\frac{\alpha_{n-1}^{(n)}}{2}}{\nu} - r\alpha_{n-1}^{(n)} > 0$, or, equivalently, $\nu B - D - \alpha_{n-1}^{(n)}(\frac{\nu + 2r\nu - 3}{2}) > 0$, which is impossible as $\nu B - D < 0$, $r \geq \frac{3-\nu}{2\nu}$, and $\alpha_{n-1}^{(n)} > 0$.

Thus, $\alpha_n^{(n)}$ satisfies $B + \alpha_n^{(n)} - \frac{D - \alpha_{n-1}^{(n)} - (1-\nu)\frac{\alpha_n^{(n)} + \alpha_{n-1}^{(n)}}{2}}{\nu} - r(\alpha_{n-1}^{(n)} - \alpha_n^{(n)}) = 0$. Following (29) and (30), $\alpha_{n-1}^{(n)} = \alpha_n^{(n)} + \gamma^{n-1}(1-\alpha_1^{(n)}) = \alpha_n^{(n)} + (1-\alpha_n^{(n)})\frac{\gamma^{n-1}-\gamma^n}{1-\gamma^n}$. Substituting for $\alpha_{n-1}^{(n)}$ in the previous equation allows us to write $\alpha_n^{(n)}$ as

$$\alpha_n^{(n)} = \frac{2(D-\nu B) + (\nu + 2r\nu - 3)\frac{\gamma^{n-1}-\gamma^n}{1-\gamma^n}}{4 + (\nu + 2r\nu - 3)\frac{\gamma^{n-1}-\gamma^n}{1-\gamma^n}}. \tag{31}$$

(ii) Welfare in an n-tranche equilibrium

The expressions in (29), (30), and (31) fully characterize any n-tranche candidate equilibrium. However, bank competition in the first stage ensures that whichever value of n maximizes welfare is the equilibrium value of n. Therefore, we proceed by calculating welfare associated with an n-tranche candidate equilibrium.

First, the welfare loss of hidden borrowing in each tranche i is

$$(\nu r - 1)\int_{\alpha_i^{(n)}}^{\alpha_{i-1}^{(n)}} (\alpha_{i-1}^{(n)} - x)dx = \frac{\nu r - 1}{2}\left(\alpha_i^{(n)} - \alpha_{i-1}^{(n)}\right)^2. \tag{32}$$

It follows that the total welfare loss of hidden borrowing in the n-tranche equilibrium is

$$\begin{aligned}\frac{r\nu - 1}{2}\sum_{i=0}^{n-1}(\alpha_i^{(n)} - \alpha_{i+1}^{(n)})^2 &= \frac{r\nu - 1}{2}\sum_{i=0}^{n-1}\gamma^{2i}(1-\alpha_1^{(n)})^2 \\ &= \frac{r\nu - 1}{2}\frac{1+\gamma^n}{1-\gamma^n}\frac{1-\gamma}{1+\gamma}(1-\alpha_n^{(n)})^2. \end{aligned}\tag{33}$$

Thus, overall, the welfare associated with an n-tranche candidate equilibrium is given by

$$W(n) = \int_{\alpha_n^{(n)}}^1 (\nu B - D + 2x)dx - \frac{r\nu - 1}{2}\frac{1+\gamma^n}{1+\gamma}\frac{1-\gamma}{1-\gamma^n}(1-\alpha_n^{(n)})^2, \tag{34}$$

where the first term corresponds to the aggregate NPV of projects undertaken and the second term to the loss associated with hidden borrowing.

Note that following (31)

$$\frac{d\alpha_n^{(n)}}{dn} = (\ln\gamma)\frac{2\gamma^{n-1}(1-\gamma)(B\nu - D + 2)(\nu + 2r\nu - 3)}{(\gamma^n - \gamma^{n-1}\nu + 3\gamma^{n-1} + \gamma^n\nu - 2r\gamma^{n-1}\nu + 2r\gamma^n\nu - 4)^2} < 0, \tag{35}$$

where the inequality follows as: $(B\nu - D + 2)$ is positive by assumption, in this region $r > \frac{3-\nu}{2\nu}$, so $(\nu + 2r\nu - 3) > 0$, $\gamma < 1$, and thus, $\ln\gamma < 0$. Furthermore, note, that as $n \to \infty$, $\alpha_n^{(n)} \to \frac{D-\nu B}{2}$. Finally, $\int_A^1(\nu B - D + 2x)dx$ is decreasing in A for $1 > A > \frac{D-\nu B}{2}$ (reflecting that above the point of efficient liquation, additional surplus is generated by increasing the range of projects undertaken) and because $\frac{d\alpha_n^{(n)}}{dn} < 0$, it follows that $\int_{\alpha_n^{(n)}}^1(\nu B - D + 2x)dx$ is increasing in n.

Consequently, a sufficient condition for $W(n)$ to be increasing in n is that $\frac{1+\gamma^n}{1-\gamma^n}(1-\alpha_n^{(n)})^2$ is decreasing in n. We establish that

$$\begin{aligned}\frac{d}{dn}\left(\frac{1+\gamma^n}{1-\gamma^n}(1-\alpha_n^{(n)})^2\right) &= \frac{d}{dn}\left(\frac{4(2-D+\nu B)^2(1-\gamma^{2n})}{(4(1-\gamma^n)+(\nu+2r\nu-3)(\gamma^{n-1}-\gamma^n))^2}\right) \\ &= 8\,(\ln\gamma)\,(2-D+\nu B)^2\,\frac{4\gamma^n(1-\gamma^n)-\gamma^{n-1}(\nu+2r\nu-3)(1-\gamma)}{(\gamma^{n-1}(\nu+2r\nu-3)(1-\gamma)+4(1-\gamma^n))^3} \\ &< 0,\end{aligned} \tag{36}$$

where the last inequality follows from substitution for γ in the numerator of the fraction to obtain $4\gamma^n(1-\gamma^n)-\gamma^{n-1}(\nu+2r\nu-3)(1-\gamma) = 2\left(\frac{\nu+2r\nu-3}{1-\nu}\right)^n\left(\nu+r\nu-2\left(\frac{(\nu+2r\nu-3)}{1-\nu}\right)^n\right)$, noting that $\nu+r\nu-2\left(\frac{(\nu+2r\nu-3)}{1-\nu}\right)^n$ increases in n and that at $n=1$, $\nu+r\nu-2\left(\frac{(\nu+2r\nu-3)}{1-\nu}\right) = \frac{(\nu+3)(2-\nu-r\nu)}{1-\nu} > 0$.

This result establishes that $W(n)$ increases in the number of tranches n.

(iii) Characterizing the limiting equilibrium

The expressions in (29), (30), and (31) fully characterize an n-tranche equilibrium. Taking the limit as $n\to\infty$, $\alpha_n^{(n)} \to \frac{D-\nu B}{2}$. Thus, (30) can be written as $1-\frac{D-\nu B}{2} = (1-\alpha_1)\frac{1}{1-\gamma}$, and thus,

$$\alpha_1 = 1 - \frac{2\,(2-\nu-r\nu)}{1-\nu}\left(1-\frac{D-\nu B}{2}\right). \tag{37}$$

Substituting this expression into (30) and substituting for γ we obtain

$$\alpha_i = 1 - \frac{2-r\nu-\nu}{1-\nu}\left(1-\frac{D-\nu B}{2}\right)\frac{1-\left(\frac{\nu+2r\nu-3}{1-\nu}\right)^i}{1-\frac{\nu+2r\nu-3}{1-\nu}}, \tag{38}$$

which completes the characterization in the statement of the proposition.

(iv) Existence

i) Description of strategies and beliefs

The above discussion demonstrates that along the equilibrium path, contracts must take the form described in the proposition. We have not fully described the equilibrium strategies and thus do so now:

For any bank at $t=0$, offer the equilibrium contract specified in the proposition in any round of the competitive process. For any bank in the interim period $t=1$, offer to any borrower who holds an existing (p,q) repayment schedule a contract with first-period payment $p' = \max\{\alpha|\ \alpha$ adopts the (p,q) contract$\}$ and q' set at the break-even rate, given p' and the set $\{\alpha|\ \alpha$ adopts the (p,q) contract$\}$.

In $t=0$, borrowers entertain new offers until a bank offers a contract that delivers a level of utility not less than that described in the proposition. A borrower of type α chooses a schedule (p,q) that would maximize her utility if she held that schedule until $t=2$, or that minimizes hidden borrowing if several schedules deliver the same level of utility. In the interim period, a borrower, starting with schedule (p,q), continues to entertain new offers until a bank offers a contract that delivers a level of utility not less than that of a contract with first-period payment $p' = \max\{\alpha|\ \alpha$ adopts the (p,q) contract$\}$ and q' set at the break-even rate, given p' and the set $\{\alpha|\ \alpha$ adopts the (p,q) contract$\}$.

Along the equilibrium path, beliefs are consistent with rational expectations. Off-equilibrium, borrowers have passive beliefs, that is, they continue to expect that all other banks will make equilibrium offers after they receive a deviating offer. Both on- and off-equilibrium banks in the interim period believe that each borrower has taken the schedule from the available contract that maximizes her utility as if she were planning to hold the schedule until $t = 2$ (or, in the where case the borrower is indifferent, chose the contract that minimizes hidden borrowing).

These proposed strategies and beliefs guarantee that the first bank offer in $t = 0$ is accepted.

ii) Optimality of strategies

Having fully specified strategies and beliefs, we proceed by checking that there are no deviations that attract borrowers without incurring losses for any of the agents at any stage of the game.

First, note that it is clear, from the way that borrower strategies are specified that borrowers are optimizing, given their expectations of bank offers.

In the interim period, banks are optimizing, as any other offer that would attract borrowers would entail losses. To see this result, consider a pool of borrowers (a, b) and suppose that a rival bank in the interim period offers a contract (p', q') with $p' \neq b$.

First, suppose that $p' > b$. Following a logic similar to the proof of Proposition 1, this deviation would either attract no borrowers or entail losses for the deviating bank. In particular, if one borrower is attracted by the (p', q') offer, then all borrowers in the pool are attracted by the offer. This would require q' to be below the break-even level and thus entail losses.

Next, suppose that $p' < b$. As in the proof of Proposition 1, following Lemma 1, the interim deviation either attracts all borrowers, or attracts lower type borrowers. We consider each case in turn and argue:

1) There is no interim deviation that attracts all borrowers and is profitable.

2) There is no interim deviation that attracts some borrowers, but not all, and is profitable.

1) There is no interim deviation that attracts all borrowers and is profitable.

As the new contract attracts all borrowers, it attracts b-type borrowers, and thus, $(b - p') + \nu(B + b - q') > \nu(B + b - q)$. Equivalently, $\frac{(b-p')}{\nu} + q > q'$. For the deviation to be profitable, $q' \geq \frac{D-p'-(1-\nu)\frac{(a+b)}{2}}{\nu}$. Substituting for q and combining the above two inequalities, $\frac{(b-p')}{\nu} + \frac{D-b-(1-\nu)\frac{(a+b)}{2}}{\nu} > q' \geq \frac{D-p'-(1-\nu)\frac{(a+b)}{2}}{\nu}$, which cannot be true because the two sides of the inequality are equal, producing a contradiction. Thus, there is no profitable deviation that attracts all borrowers in the pool.

2) There is no deviation that attracts some borrowers but not all and is profitable.

Following Lemma 1, since $p' < b$ higher-type borrowers choose the original contract and lower-type borrowers choose the new contract.

The borrower b' who is indifferent between both contracts does not borrow from the hidden lender under the new contract because b' lies between α and β, and neither of these types borrow from the hidden lender under the new contract. The indifferent borrower b' is therefore characterized by $\nu(B + b' - q - r(p - b')) = (b' - p') + \nu(B + b' - q')$, which implies that $q - \frac{(1-r\nu)}{\nu}(p - b') = q'$.

Suppose that the new contract is profitable. Then, $q' \geq \frac{D-p'-(1-\nu)\frac{(a+b')}{2}}{\nu}$. Substituting for $q' = q - \frac{(1-r\nu)}{\nu}(p-b')$ in this inequality and substituting for $q = \frac{D-b-(1-\nu)\frac{(a+b)}{2}}{\nu}$ and $p = b$, it follows that $\frac{D-b-(1-\nu)\frac{(a+b)}{2}}{\nu} - \frac{(1-r\nu)}{\nu}(b-b') \geq \frac{D-p'-(1-\nu)\frac{(a+b')}{2}}{\nu}$, or, equivalently, $p' - b > (\frac{(1-\nu)}{2} + (1-r\nu))(b-b')$. The left side of this inequality is negative, as $b > p'$, and the right side is positive, as $r > \frac{3-\nu}{2\nu}$ and $b > b'$, producing a contradiction. Thus, the new contract cannot be profitable.

Thus, there is no profitable deviation at period $t = 1$.

Finally, turning to the strategies of the banks at $t = 0$, note that any deviation at $t = 0$ that does not conform with Proposition 1 would be competed away by contracts that are consistent with these results, as they rely only on period 1 competition. Therefore, we need only restrict our attention to the type of feasible n-tranche offers that are characterized by (29), (30), and (31). Given that the proposed equilibrium maximizes the borrower's expected utility at $t = 0$ within this class, there cannot be an offer that is both profitable and preferred by borrowers. This exhausts all possible deviations at all possible stages of the game by both borrowers and banks and demonstrates that this point is indeed an equilibrium.

(v) Uniqueness

Expressions (29), (30), and (31) characterize an n-tranche equilibrium, with any indifference resolved through the tie-breaking assumptions. These expressions are a consequence of the indifference conditions of borrowers and the competition of banks at $t = 1$; thus, they hold for any candidate equilibrium or deviation that can be considered at $t = 0$. Given that borrowers do not know their types at $t = 0$, competition between banks implies that the n-tranche contract that maximizes ex-ante welfare is the only possible one in equilibrium.

There are other equilibria that may induce preliminary rounds of offers that get rejected, or that involve alternative off-equilibrium beliefs, but these are inconsequential for equilibrium outcomes. ■

Proof of Corollary 2

Proof. It is immediate that

$$\frac{d\alpha_i}{dr} = (1 - \frac{D - \nu B}{2})i\nu \frac{\left(\frac{(\nu+2r\nu-3)}{1-\nu}\right)^i}{\nu + 2r\nu - 3} > 0.$$

■

Proof of Corollary 3

Proof. In the range $r > \frac{2-\nu}{\nu}$, welfare is constant and equal to

$$W = \int_{\frac{D-\nu B}{2}}^{1} (\nu B - D + 2x)dx = \left(1 - \frac{D - \nu B}{2}\right)^2.$$

In the range $\frac{2-\nu}{\nu} \geq r > \frac{3-\nu}{2\nu}$,

$$\begin{aligned} W &= \lim_{n\to\infty} (1 - \alpha_n^{(n)})(\alpha_n^{(n)} + 1 - (D - \nu B)) - \frac{r\nu - 1}{2} \frac{1 + \gamma^n}{1 + \gamma} \frac{1 - \gamma}{1 - \gamma^n} (1 - \alpha_n^{(n)})^2 \\ &= (1 - \frac{D - \nu B}{2})(\frac{D - \nu B}{2} + 1 - (D - \nu B)) - \frac{r\nu - 1}{2} \frac{1 - \frac{\nu+2r\nu-3}{1-\nu}}{1 + \frac{\nu+2r\nu-3}{1-\nu}} (1 - \frac{D - \nu B}{2})^2 \\ &= \frac{1 + r}{2}\nu \left(1 - \frac{D - \nu B}{2}\right)^2. \end{aligned}$$

Trivially, in this range, welfare is monotonically increasing in r.

Finally, in the range $\frac{3-\nu}{2\nu} \geq r \geq \frac{1}{\nu}$, we have

$$
\begin{aligned}
W &= \int_{\frac{\nu+2D-2B\nu+2r\nu-3}{\nu+2r\nu+1}}^{1} (\nu B - D + 2x)dx - (\nu r - 1) \int_{\frac{\nu+2D-2B\nu+2r\nu-3}{\nu+2r\nu+1}}^{1} (1-x)dx \\
&= 8\nu \frac{r+1}{(\nu + 2r\nu + 1)^2} \left(1 - \frac{D - \nu B}{2}\right)^2 .
\end{aligned}
$$

Note that at $r = \frac{2-\nu}{\nu}$, $\frac{1+r}{2} = 1$ and at $r = \frac{3-\nu}{2\nu}$, $8\nu\frac{r+1}{(\nu+2r\nu+1)^2} = \frac{1+r}{2}\nu = \frac{3+\nu}{4}$, so that there is a smooth-pasting of welfare through the different ranges.

Trivially, W is constant when $r > \frac{2-\nu}{\nu}$ and increasing in r when $\frac{2-\nu}{\nu} > r > \frac{3-\nu}{2\nu}$. In the range $\frac{3-\nu}{2\nu} > r \geq \frac{1}{\nu}$; then, $\frac{dW}{dr} = 8\nu\left(1 - \frac{D-\nu B}{2}\right)^2 \frac{1-3\nu-2r\nu}{(1+\nu+2r\nu)^3}$, which is negative when $r > \frac{1-3\nu}{2\nu}$. Note that as $\nu > 0 > -\frac{1}{3}$, it follows that $\frac{1}{\nu} > \frac{1-3\nu}{2\nu}$. Thus, throughout the range $\frac{3-\nu}{2\nu} > r \geq \frac{1}{\nu}$, $r > \frac{1-3\nu}{2\nu}$, and thus, $\frac{dW}{dr} < 0$. ■