

# Targeted product design: Locating inside the Salop circle<sup>1</sup>

Heski Bar-Isaac  
U Toronto

Guillermo Caruana  
CEMFI

Vicente Cuñat  
LSE

January, 2014.

## Abstract

Product design is a key choice for firms. We consider the trade-off associated with well-targeted designs that are much more suitable for some types of consumers against more generic designs that are unremarkable and inoffensive to all types. We introduce a model that adapts the familiar Salop circle model (1979) by allowing firms to locate on the interior of the circle, thereby we allow for continuous design choices between the extremes of fully targeted and fully generic designs. We provide simple sufficient conditions that ensure extreme or interior design choices. These conditions on consumer preferences apply both to a monopolist and to competing firms under two different models of competition.

---

<sup>1</sup>We thank Haritz Garro and Mariano Irace for superb research assistance, and David Myatt and Justin Johnson for helpful discussions.

Contact info: Bar-Isaac: heski.bar-isaac@rotman.utoronto.ca; Rotman School of Management, University of Toronto, 105 St George St, Toronto, ON M5S 3E6, Canada; Caruana: caruana@cemfi.es; Casado del Alisal 5, 28014 Madrid, Spain; and Cuñat: v.cunat@lse.ac.uk; The London School of Economics, Houghton Steet, WC2A 2AE, London, UK.

# 1 Introduction

Firms constantly make decisions not only about prices and quantities, but also regarding the kind of goods that they produce. Even though some choices of product characteristics may be costless to the firm, they are non-trivial decisions, as making a product more attractive to some consumers may make it less attractive to others.

Our focus in this paper is the trade-off inherent in the targeting decision associated with product design. A more targeted design satisfies the consumers to whom it is directed but at the cost of alienating other types of consumer. Instead, more generic products might alienate few consumers but are unlikely to excite the passions of any. Examples of this trade-off in the choice between more targeted and more generic designs are wide-ranging. Specialized bicycles can be targeted towards racers, mountain-bikers, or commuters, for example; while hybrid models may try to appeal a broader range of customers without fully satisfying any of them. Restaurants can choose very authentic tailored cuisines or offer more bland or less daring offerings. Software designers might choose to design a very slick clean program to address a specific need, or slower more cumbersome designs that can handle many uses. Even in the choice of color, a fashion or product designer might choose a specific bright color that may appeal to some, or a more neutral palette that may not thrill any consumer, but is less likely to offend anyone.

To address the issue of how much to target a design rather than choose more generic designs that are neither loved nor loathed, we introduce a new model. In our model designs lead to demand rotations (discussed in particular in Johnson and Myatt, 2006). A relatively more generic design leads to a rotation of a firm's demand curve, whereby the consumer who enjoys the good most gains less utility from a more generic design, but the consumer who enjoys it the least gains more utility.<sup>2</sup> This design decision thus con-

---

<sup>2</sup>Further, formally a rotation imposes a little more structure in imposing that the demand curves associated with different designs cross only once.

trasts from standard models of horizontal differentiation, where the concern is which consumers to satisfy and which to alienate, and models of vertical differentiation where designs are commonly ranked in consumers' preferences.

As an example, consider a new Persian restaurant. It is naturally limited in the range that it can offer, and so must choose between menu items that are designed with broader audiences in mind (offering French fries instead of rice, or include burgers on the menu) or items that might appeal only to more refined palettes (such as *kalleh pacheh*—a traditional broth prepared with lamb's head and trotters). This kind of design choice creates an interesting trade-off since a blander, more conventional menu might appeal to a broader audience but at the same time means that no individual diner is likely to be so enamored with the cuisine that the restaurateur can charge a very high price. In seeking to attract a wider range of horizontal preferences through the design and reducing the dispersion in the valuations of different consumers, there is a sense in which there is a vertical quality drop through the loss of authenticity. Similarly, a software designer in seeking to broaden appeal by adding new features (thereby ensuring less dispersion in the valuations of different consumers) might create a slower running program or a more complex, less intuitive and bloated interface.

In considering design, the restaurateur, software designer, or firm, more generally, must consider the underlying consumer preferences. In particular, a key determinant of design will be the extent to which the breadth of appeal increases while the product suffers some fall in vertical quality for the aficionado who most appreciates the product or service. If the aficionado cares a great deal for authenticity but is relatively insensitive once moving away from a genuinely authentic cuisine, then the restaurateur will do best by choosing an extreme offering—either as bland and generic as he can be to cater to a wide audience, or as authentic as possible to target the extreme tastes. Instead if, the aficionado very much dislikes bland generic offerings but is relatively insensitive across offerings that are somewhat authentic, then

the restaurateur optimizes with an intermediate menu that balances between the tastes of aficionado and the broader population. Similarly, if adding features degrades quality or slows down software to a greater extent when there are few features than many, the software firm optimizes by offering either a very stripped down or a very broad program. Instead, in the opposite case when the effect of extra features in terms of slowing down the program or making the interface more complicated gets worse and worse, then software might optimize through an intermediate offering.

We model design choices as an intuitive trade off between conventional representations of horizontal and vertical differentiation. Formally, we adapt the Salop (1979) model where consumers are located on the circumference of a circle to allow firms to locate on the interior rather than only on the edge of the circle. Locations closer to the center of the circle correspond to more generic offerings that appeal to a broad base and locations close to the circumference are targeted niche offerings. Consumer preferences are reflected in horizontal costs associated with moving around the circle and vertical costs associated with moving to the interior, thus a generic offering involves relatively high vertical costs for all consumers, but also lower horizontal costs. We abstract from costs to the producer associated with different designs, though these can be incorporated in a straightforward way.

We find a number of results. Firstly, we find sufficient conditions that ensure extreme product offerings—that is, offerings that are either as generic or as targeted as possible. As suggested above, these key conditions are related to the concavity or convexity in the vertical costs associated with consumer preferences for products on the interior. We show that this result holds for the monopoly case and various straightforward forms of competition. Secondly, we show that the higher the marginal cost of production, the more targeted the offering. The intuition is a familiar one—a firm with a very high marginal cost must charge a relatively high price and so values variance in consumer valuations in the hopes of finding some consumers willing to buy;

whereas a firm charging a relatively low price expects most consumers would be disposed to purchase unless the good is a very poor match, so that the firm benefits from reducing variance by choosing to be more generic. Although we begin by outlining a monopoly model, this result applies to monopolistic competition, and allows us to show different market configurations in terms of product design.

In analyzing demand rotations, this paper is related to a recent literature in economics and marketing that has explored information disclosure in monopoly and competitive settings. Anand and Shachar (2011), for example, demonstrate that for television shows, providing information through advertisements can decrease the demand for some consumers while increasing it for others. Earlier theoretical work by Lewis and Sappington (1991, 1994) consider firms' incentives to provide consumers (of two possible types) with private information.<sup>3</sup> More recently, Johnson and Myatt (2006) provide a general treatment allowing for a continuum of consumer types and introducing demand rotations—that is families of demand curves where any pair of demand curves cross only once—and consider several examples that ensure an ordering of demand curves that leads a monopolist to an extremal choice.<sup>4</sup> Kuksov and Lin (2010), Gu and Xie (2012), Sun (2010) and Sun and Tyagi (2012) consider the incentives of different types of firms to provide different kinds of information (for example, on vertical product quality as opposed to information on individual product fit).<sup>5</sup> Similar, to our result on product design, high marginal cost firms often have greater incentives to provide more (idiosyncratic) product fit information.

While the literature has tended to focus on information provision, the

---

<sup>3</sup>Thus, this paper and the literature considered below proceeds on a different course to Ottaviani and Prat (2001) that considers provision of public information.

<sup>4</sup>See also Ganuza and Penalva (2010) who provide an ordering of informative signals.

<sup>5</sup>Another stream of literature allows for firms to release entirely general signals of information rather than impose some structure on the signals or require that information is either disclosed or not. See, for example, Saak (2006), Kamenica and Gentzjow (2011), Rayo and Segal (2010), and Boleslavsky, Cotton and Gurnani (2013).

work on the role of product design in inducing demand rotations is more limited. In particular, Kuksov (2004) considers a binary design choice, and Johnson and Myatt (2006), Larson (2011), and Bar-Isaac, Caruana and Cuñat (2012) make assumptions that ensure only two designs (the extreme generic or niche designs) arise. In a framework more closely related to ours, von Ungern-Sternberg (1988) instead imposes conditions that ensure an interior solution, and analyses a symmetric free-entry equilibrium, where the “vertical” cost that leads to a trade-off is not one that affects consumer utility directly but instead raises the firm’s cost of production.

In providing a clear and intuitive representation, building off the familiar Salop (1979) model, our framework makes clear the trade-off between reducing horizontal differentiation and reducing vertical quality that is inherent in demand rotations, and provides a simple and flexible framework for considering this trade-off. Our results highlight that either extreme or interior designs might be optimal and provide simple sufficient conditions for either. Further, we show that firms with higher marginal cost of production choose more targeted designs. In analyzing both a monopoly model and extensions to different forms of competition, we demonstrate the robustness of these results.

## 2 A Model of Design: Monopoly

Consider a ring of outer-radius 1 and inner-radius  $B$  with  $B > 0$ . Locations anywhere in this ring correspond to different possible designs.<sup>6</sup> A mass of  $2\pi$  consumers is uniformly distributed on the outer circle. A monopolist, with a constant per unit marginal cost  $m$ , can locate anywhere within the ring.<sup>7</sup> If the firm locates exactly at the location of a consumer, this consumer

---

<sup>6</sup>For minor technical reasons, the algebra in this paper is written for  $B > 0$ . The results for the  $B = 0$  case can be analyzed as the limit case as  $B$  tends to 0.

<sup>7</sup>While we consider variation in marginal costs, the results apply equally to varying quality. In this interpretation one could think of an unchanging marginal cost and a low

values the product  $V$ . Otherwise, the consumer must incur travel costs to reach the firm. She first travels along a radius towards the center of the ring and, only then, travel along the arc.<sup>8</sup> If she travels a distance  $y$  along the radius and  $x$  along the arc, the travel costs are assumed to be  $c(y) + x$  with  $c(\cdot)$  twice continuously differentiable and  $c'(\cdot) > 0$ . That is, we assume linear unit travel costs along the arc, but allow any increasing shape for the cost of travelling along a radius.<sup>9</sup> By construction, the cost of traveling along a radius is common to all consumers and can be interpreted as vertical differentiation. Meanwhile, the cost of traveling along the arc varies across consumers depending on their locations. Thus, a change in this transport cost can be interpreted as a change in horizontal differentiation. Throughout the paper we refer to the cost of traveling along the arc as a horizontal cost and the cost of traveling along a radius as a vertical cost. A central element of the model is that firm strategies always involve a trade-off between these

---

$m$  would correspond to a higher quality delivered.

<sup>8</sup>We impose that the consumer travels towards the center and along a ring independently, and allow for different costs. Hence these dimensions are better suited for two different characteristics of a good (such as the brightness and hue of its colour) rather than dimensions in a physical space.

<sup>9</sup>Linear costs along the arc restrict the analysis to linear demand functions, while unit costs are without loss of generality.

two costs. This is all illustrated in the figure below:

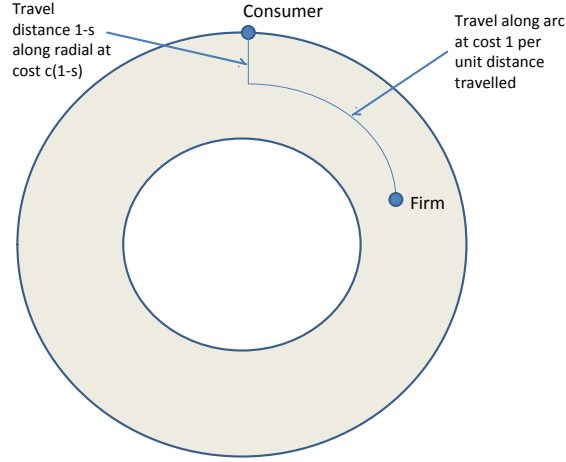


Figure 1: Design and consumer travel costs

Without loss of generality the firm is located at angle 0. Thus, the location decision boils down to choosing how far inside the ring it wants to be, which we capture by  $s \in [B, 1]$ . Locating at  $s = 1$  corresponds to a fully tailored design in which the firm aims for a niche consumer base. Such a design maximizes the valuation of the consumer located at angle 0, but it also maximizes the dispersion of valuations and, in particular, it minimizes the appeal of the product for the consumer located at angle  $\pi$ . Locating closer to the centre, reduces the heterogeneity of consumer valuations and has a similar effect to a reduction of horizontal transport costs in a standard circular setting. However, moving towards the center also reduces the vertical quality of the good by imposing a common additional cost to all consumers. The most general design possible corresponds to  $s = B$ ; while still aiming to a particular type of consumer it remains as broad as possible, minimizing the dispersion of consumer valuations.

If a monopolist chooses a price  $p$  and a design  $s$ , the marginal consumers



who are indifferent between purchasing or not are located at angles  $x$  and  $-x$ , where  $x$  satisfies:

$$V - c(1 - s) - sx - p = 0.$$

Thus, the demand for a monopoly who chooses price  $p$  and design  $s$  is given by:

$$q(p, s) = \max(0, \min(2\pi, \frac{2}{s}(V - c(1 - s) - p))).$$

## 2.1 Optimal design

We consider the optimal design choice for a monopolist who can choose both design and price. For simplicity, we assume that optimal choices lead to a demand that is an interior  $x$  in the interval  $(0, 2\pi)$  (which holds if  $V$  is above  $m$ , but sufficiently low).<sup>10</sup> In this case the demand function simplifies to:

$$q(p, s) = \frac{2}{s}(V - c(1 - s) - p) \tag{1}$$

and the monopolist's problem is to choose  $s$  and  $p$  in order to maximize:

$$\Pi(p, s) = \frac{2}{s} [V - c(1 - s) - p] (p - m). \tag{2}$$

Note first that the demand function  $q(p, s)$  is linear and that, the higher  $s$ , i.e. the more niche the design, the steeper the slope of (inverse) demand, corresponding to more diverse valuations by different consumers. A higher  $s$  also involves a higher intercept with the price axis, representing a higher valuation of the consumer who likes the good most. Thus, any two designs result in demands that cross only once, and so different design choices induce demand rotations as in Johnson and Myatt (2006).

Our assumptions on the differentiability of transport costs imply the dif-

---

<sup>10</sup>The same qualitative results are obtained for the case in which it is optimal to serve the whole circle. Note that this is the case when  $B$  tends to 0.

ferentiability of the profit function, and allow us to use first and second order conditions to establish the following results.

**Proposition 1** *When the optimal design  $s^*$  is interior it satisfies the following conditions*

$$2s^*c'(1-s^*) = V - c(1-s^*) - m \quad (3)$$

$$c''(1-s^*) > 0 \quad (4)$$

**Proof.** It is straightforward to solve for the optimal price  $p^* = \frac{V-c(1-s)+m}{2}$  and quantity  $q^* = \frac{V-c(1-s)-m}{s}$ .

By the envelope theorem we can write  $\frac{d\Pi(p,s)}{ds} = \frac{dq}{ds}(p-m)$ ; it follows that the first order condition with respect to design is equivalent to  $\frac{dq}{ds} = 0$ .

We can write

$$\frac{dq}{ds} = -\frac{2}{s^2}(V - c(1-s) - p) + \frac{2}{s}c'(1-s). \quad (5)$$

Substituting  $p$  for its optimal value  $p^* = \frac{V-c(1-s)+m}{2}$ , and rearranging gives the first condition in the statement of the proposition.

At an optimal interior design both the first order condition and the second order condition must be satisfied. The latter is given by:

$$\begin{aligned} \frac{d^2\Pi(p,s)}{ds^2} &= -\frac{p-m}{s^2}(2c'(1-s) - q(p,s)) + \frac{p-m}{s}(-2c''(1-s) - 2c'(1-s) + q(p,s)) \\ &< 0. \end{aligned}$$

Note that when first order condition is satisfied then  $2c'(1-s) - q(p,s) = 0$ , so that the second order condition simplifies to

$$\frac{d^2\Pi(p,s)}{ds^2} = -2\frac{p-m}{s}c''(1-s) < 0,$$

or, equivalently, the second condition in the statement of the proposition. ■

The following corollaries are immediate.

**Corollary 1** *A necessary condition for an interior design solution is that a consumer's vertical transport cost is locally convex.*

**Corollary 2** *If vertical transport costs are concave—that is  $c''(x) < 0$  for all  $x$ —then a monopolist optimally chooses an extremal design  $s^* \in \{B, 1\}$ .*

We provide some intuition for these results. Consider the different demand curves that are traced out as the monopolist chooses different designs. A concave travel cost  $c(\cdot)$  ensures that as, the monopolist moves from niche designs that induce steep demand functions to flatter broad designs the drop-off in the price intercept is not too severe. In particular, the upper envelope of what can be achieved by the family of demand rotations is traced out by the most niche and the most broad designs. Thus, the monopolist chooses one of these two designs.

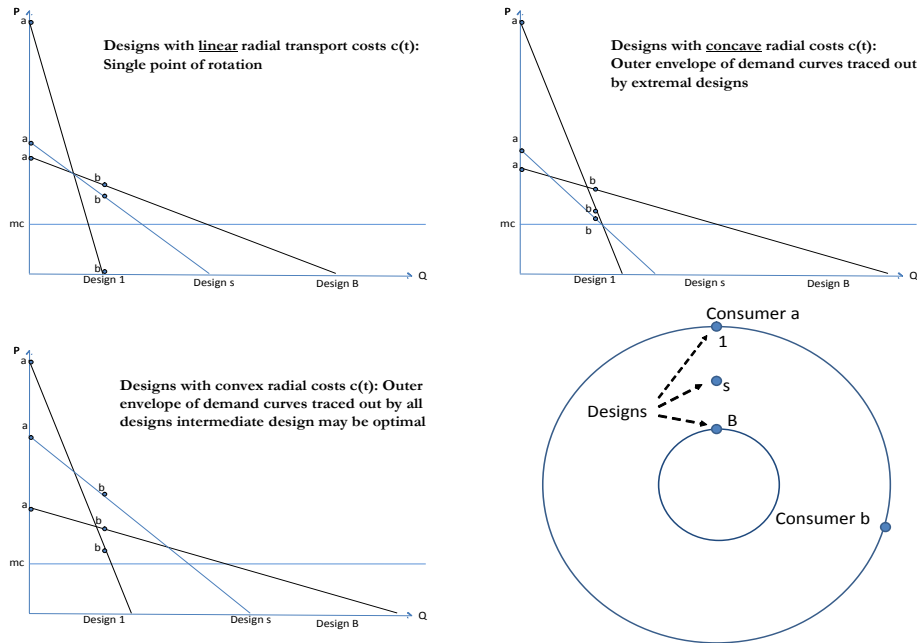


Figure 2: Summary of rotation orderings as a function of transport costs

The result is intuitive. Demand rotations imply that broader designs entail a vertical cost to all consumers. The concavity of  $c(\cdot)$  means that as a firm gets broader, the additional vertical cost grows slower than the reduction in horizontal costs. This means that once the firm finds it profitable to choose a broader design, it will be more profitable to choose the broadest design possible. Note that the converse to Proposition 1 is not true. That is, if  $c(\cdot)$  is convex, it is not guaranteed that the optimal design is going to be an interior one. In other words, firm profits can be quasi-convex in design even if the cost of travelling along a radial is convex. This can be easily perceived by explicitly looking at the first order conditions for the profit maximization program described in (2).

Note that while expressions (3) and (4) characterize a local optimum, they do not guarantee that the solution is globally optimal. We can, however, establish elementary conditions that are sufficient to guarantee that an interior design is optimal.

**Proposition 2** *An interior optimal design arises if*

$$2Bc'(1-B) + c(1-B) > V - m > 2c'(0)$$

**Proof.** The firm necessarily prefers an interior solution if the objective function (2) satisfies  $\Pi'(1, p^*(1)) < 0$  and  $\Pi'(B, p^*(B)) > 0$ . Substituting  $p^* = \frac{V - c(1-s) + m}{2}$  into (2) allows us to write profits as a function of design alone:

$$\Pi(s) := \frac{2}{s} \left( \frac{V - c(1-s) - m}{2} \right)^2. \quad (6)$$

Given that

$$\Pi'(s) = -\frac{(V - c(1-s) - m)^2}{2s^2} + \frac{c'(1-s)}{s} (V - c(1-s) - m),$$

we can write

$$\begin{aligned}\Pi'(1) < 0 &\Leftrightarrow V - m > 2c'(0) \\ \Pi'(B) > 0 &\Leftrightarrow 2Bc'(1 - B) + c(1 - B) > V - m,\end{aligned}$$

which concludes the proof. ■

Essentially, in order for a solution to be interior a sufficient condition is that the cost function  $c(\cdot)$  is sufficiently flat at  $y = 0$  and steep enough at  $y = 1 - B$ . While these two conditions may not be always satisfied, they are interesting for two reasons. First, they are simple to check and interpret, and second, they do not impose any particular functional behavior in the interior of the domain, in particular whether the function needs to be concave or convex.

Next we turn to the comparative statics of the optimal design, and show that a firm with higher marginal costs would choose a more targeted, niche design.<sup>11</sup>

**Proposition 3** *A firm with higher marginal cost of production  $m$  chooses a nichier design.*

**Proof.** To prove this, it is sufficient to show that

$$\forall m_1 > m_2, \forall s_1 > s_2 \quad \Pi(s_1, m_2) > \Pi(s_2, m_2) \Rightarrow \Pi(s_1, m_1) > \Pi(s_2, m_1) \quad (7)$$

Note that  $\frac{2}{s_1} \left( \frac{V - c(1 - s_1) - m_2}{2} \right)^2 = \Pi(s_1, m_2) > \Pi(s_2, m_2) = \frac{2}{s_2} \left( \frac{V - c(1 - s_2) - m_2}{2} \right)^2$  implies that  $(\sqrt{s_1} - \sqrt{s_2}) m_2 > \sqrt{s_1}(V - c(1 - s_2)) - \sqrt{s_2}(V - c(1 - s_1))$ . Given that  $(\sqrt{s_1} - \sqrt{s_2}) m_1 > (\sqrt{s_1} - \sqrt{s_2}) m_2$  we can write  $(\sqrt{s_1} - \sqrt{s_2}) m_1 > \sqrt{s_1}(V - c(1 - s_2)) - \sqrt{s_2}(V - c(1 - s_1))$ , which implies that  $\Pi(s_1, m_1) > \Pi(s_2, m_1)$ .

---

<sup>11</sup>Equivalently, as suggested in Footnote 7, the proposition can be interpreted as showing that a firm of lower quality chooses a nichier design.

Given that  $\Pi$  is continuous in  $(s, m)$ , that the condition (7) above implies that  $\Pi$  satisfies the single crossing property in  $(s, m)$  as defined in Milgrom-Shannon (1994). Thus, this proposition is just a particular case of Theorem 4 in Milgrom-Shannon (1994), which establishes monotone comparative statics.

■

So far we have characterized the design choices of a monopolist, in what follows we embed the design model in two competitive models. First, a sequential search model of monopolistic competition, next a duopoly in which firms simultaneously compete in the two dimensional design space. We show that the monopoly results extend easily to a competitive environment, and explore the market configurations that arise in a competitive design model.

### 3 Monopolistic Competition

We can maintain the form of consumer preferences, and firm design choices, but adapt the model described above to allow for competition many firms in a simple fashion by supposing that consumers must incur search costs to observe product offerings. That is, as in Wolinsky (1986) or Anderson and Renault (1999) consumers incur a search cost  $a$  to learn both the price and utility they would obtain from a new firm (this is similar to Diamond (1971); however in Diamond's model products are homogeneous). As in Bar-Isaac, Caruana and Cuñat (2012), we adapt the supply side to suppose that in addition to choosing prices, firms can choose designs. While Bar-Isaac, Caruana and Cuñat (2012) considered a reduced form design decision, in which optimal designs arise and consider implications for firm profitability, here we consider the specific design choice outlined in Section 2, and demonstrate that the results in that section on design choice are robust to this form of competition.

We can allow for heterogeneity in firms' marginal costs of production and can index the firms in the industry by  $i \in I$  where  $I$  is the continuum of firms

that is active in the industry.<sup>12</sup> All types of firms are uniformly distributed around the circle in terms of their angle of rotation.<sup>13</sup> Further, denote the design choice of firm  $i$  by  $s_i$  and its price by  $p_i$ .

Just as in McCall (1970), if a consumer finds it worthwhile to search at all then she optimizes by choosing a threshold rule, continuing buying if obtaining a net utility greater than or equal to  $U$  (which depends on her expectations of firm strategies) and otherwise continuing to search.<sup>14</sup>

Since firms' decisions are not observed by consumers until the consumer visits the firm, then a firm deviating from its equilibrium strategy will have no effect on consumers' reservation utilities. Consequently, in an equilibrium, when a consumer visits firm  $i$  (which without loss of generality we can suppose to be located at 0), the firm anticipates selling to all consumers located between angles  $x$  and  $-x$  where  $V - c(1 - s_i) - s_i x - p_i = U$ .

Thus, the demand for firm  $i$  when choosing price  $p_i$  and design  $s_i$  is:

$$q(p_i, s_i, U) = \max(0, \min(2\pi, \frac{2}{s_i} (V - U - c(1 - s_i) - p_i))). \quad (8)$$

Note that this expression is similar to (1) but features the term  $V - U$  in place of  $V$ . The stopping rule  $U$  is determined by consumer preferences and the overall market configuration. Therefore, from the firm's perspective this is a constant which it cannot effect. Analogously to Proposition 1, it is immediate that if firm  $i$  optimizes by choosing an interior design  $s_i^*$  then this

---

<sup>12</sup>Allowing for a finite number of firms requires writing out the expressions below with summations rather than integrals. With a large number of firms the analysis is otherwise identical. With a small number of firms, the analysis requires explicitly modelling the possibility of revisiting and whether the firm has any direct influence on the consumer stopping rule.

<sup>13</sup>This can be sustained as an equilibrium of a model in which firms choose their location.

<sup>14</sup>Without loss of generality, we can consider a consumer located at 0. Firm  $i$ 's location from the consumer's perspective is uniformly distributed on  $(0, 2\pi)$ . Consequently, similar to Bar-Isaac, Caruana and Cuñat (2012), it can be shown that  $U$  is implicitly defined by

$$\int_{i \in I} \int_0^{2\pi} (V - c(s_i) - p_i - U - s \min\{x, 2\pi - x\}) dx di = a$$

design satisfies

$$2s_i^*c'(1 - s_i^*) = V - U - c(1 - s_i^*) - m_i, \text{ and} \quad (9)$$

$$c''(1 - s_i^*) > 0. \quad (10)$$

Similarly, analogously to Proposition 2, the firm necessarily optimizes by choosing an interior design as long as

$$2Bc'(1 - B) + c(1 - B) > V - U - m_i > 2c'(0).$$

Corollaries 1 and 2, and Proposition 3 apply immediately. The nature of the vertical transport cost  $c$  can therefore induce very different market configurations. Suppose, for example, that marginal costs are continuously distributed over an interval: When Corollary 2 holds, a polarized distribution of product designs arises, with firms choosing either the most niche design  $s = 0$  or the broadest one  $s = B$ . A threshold  $m$  determines which firms choose each of the two designs. By Proposition 3 we know that the firms with the lowest marginal costs  $m$  prefer the broad design, with higher sales and lower markups. Simultaneously, the least efficient firms opt for the niche design with high prices and a low probability of a sale. Similarly, when Proposition 2 holds, firms choose interior designs that are distributed along one or several intervals according to each firm's marginal cost; again with more efficient firms preferring broader designs. These market configurations also have straightforward implications for prices and quantities sold. From the expressions in Proposition 1, it follows that keeping design fixed, higher marginal costs are associated with higher prices and lower sales. Moreover, the endogenous design choices reinforce these effects, as higher marginal costs induce firms to choose more specific designs that, in turn induce higher prices and lower sales. Therefore, regardless of the market configuration, prices are monotonically increasing and sales are monotonically decreasing as design moves from being more generic to more targeted. When design choices jump



discretely, so do prices and sales.

## 4 Bertrand Duopoly

We now turn to consider competition between two firms,  $i = 1, 2$ , with marginal costs of production  $m_i \geq 0$ . Consumers are interested in one product only. We assume that  $V$  is sufficiently high to guarantee full market coverage; that is, that all consumers buy from one or other of the two firms. Further, we assume that, with respect to the horizontal dimension, firms are located opposite to each other (Firm 1 at angle 0, and Firm 2 at angle  $\pi$ ).

In contrast to Section 3, we suppose that consumers can costlessly observe locations, prices and designs of both firms. We denote  $x_{12}$  to be the consumer that is indifferent between buying from Firm 1 or Firm 2. When  $x_{12} \in [0, \pi]$ , that is, when both firms are selling to some consumers, this indifferent consumer is characterized by:

$$V - c_1 - s_1 x_{12} - p_1 = V - c_2 - s_2(\pi - x_{12}) - p_2.$$

Thus, the indifferent consumer,  $x_{12}$ , can be written as:

$$x_{12} = \frac{c_2 - c_1 + p_2 - p_1 + \pi s_2}{s_1 + s_2} \quad (11)$$

More generally, the demand of firm one  $x$  is determined by

$$x = \min(\pi, \max(0, x_{12})),$$

which also encompasses the case in which one firm takes the whole market by limit pricing its rival. In the following analysis we concentrate on the case in which both firms are active in the market; that is, when  $x_{12} \in (0, \pi)$ .

We can write down firms' profits as

$$\begin{aligned}\Pi_1 &= 2x(p_1 - m_1) \\ \Pi_2 &= 2(\pi - x)(p_2 - m_2).\end{aligned}$$

Next, we analyze two different scenarios, depending on the timing assumed. For convenience, and to suppress notation, from now we refer to the vertical cost function of firm  $i$  simply by  $c_i$ , that is  $c_i = c(1 - s_i)$

**Simultaneous Timing** We start with the case in which both firms simultaneously choose their design and price. An equilibrium in this market is a vector of prices and designs  $(p_i^*, s_i^*)$  for each firm  $i = 1, 2$ , such that each firm is best responding against each other.

**Proposition 4** *A necessary condition for an interior design in the duopoly setting with simultaneous decisions is that the vertical transport cost are locally convex. Moreover, if these costs are concave, then both firms choose an extremal design.*

**Proof.** We analyze with out loss of generality the case of Firm 1. Its profit function is:

$$\Pi(p, s) = 2(p_1 - m_1) \frac{c_2 - c_1 + p_2 - p_1 + s_2\pi}{s_2 + s_1}$$

and its FOC and with respect to its design  $s_1$  is:

$$\frac{d\Pi(p, s)}{ds} = \frac{2(p_1 - m_1)}{(s_2 + s_1)^2} (c_1'(s_2 + s_1) - (c_2 - c_1 + p_2 - p_1 + s_2\pi))$$

Now one can see the SOC (in which we substitute out the FOC) delivers the desired result:

$$\begin{aligned} \frac{d^2\Pi(p, s)}{ds^2} &= \frac{2(p_1 - m_1)}{(s_2 + s_1)^2} \left( -c_1''(s_2 + s_1) - 2 \frac{(c_1'(s_2 + s_1) - (c_2 - c_1 + p_2 - p_1 + s_2\pi))}{(s_2 + s_1)} \right) \\ &\stackrel{FOC=0}{=} \frac{-2(p_1 - m_1)}{(s_2 + s_1)^2} c_1''(s_2 + s_1). \end{aligned}$$

■

**Sequential timing** We next analyze the case in which firms first choose their design simultaneously. These decisions then become public, and on a second stage, firms simultaneously choose prices.

First, consider the price decision for any given designs. We can calculate optimal prices through the FOCs

$$\begin{aligned} \frac{c_2 + m_1 - c_1 - 2p_1 + p_2 + \pi s_2}{s_1 + s_2} &= 0 \\ \frac{c_1 + m_2 - c_2 - 2p_2 + p_1 + \pi s_1}{s_1 + s_2} &= 0. \end{aligned}$$

These imply that

$$\begin{aligned} p_1^* &= \frac{c_2 - c_1 + 2m_1 + m_2 + \pi s_1 + 2\pi s_2}{3}, \text{ and} \\ p_2^* &= \frac{c_1 - c_2 + 2m_2 + m_1 + \pi s_2 + 2\pi s_1}{3}. \end{aligned}$$

Next, we solve the first stage game in which designs are chosen. Substituting both equilibrium prices in (11) yields:

$$x_{12} = \frac{1}{3} \frac{c_2 - c_1 - m_1 + m_2 + \pi s_1 + 2\pi s_2}{s_1 + s_2}.$$

Consequently, Firm 1's profits as a function of  $s_1$  and  $s_2$  can be written as

$$\Pi_1(s) = \frac{2}{9} \frac{(c_2 - c_1 - m_1 + m_2 + \pi s_1 + 2\pi s_2)^2}{s_1 + s_2}.$$

It follows that Firm 1's optimal design (when interior) satisfies

$$\begin{aligned} \frac{d\Pi_1(s)}{ds_1} &= \frac{4}{9} \frac{(c_2 - c_1 - m_1 + m_2 + \pi s_1 + 2\pi s_2)(c'_1 + \pi)}{s_1 + s_2} - \frac{2}{9} \frac{(c_2 - c_1 - m_1 + m_2 + \pi s_1 + 2\pi s_2)^2}{(s_1 + s_2)^2} \\ &= 2x_{12} \left( \frac{2}{3} (c'_1 + \pi) - x_{12} \right) = 0; \end{aligned}$$

equivalently,  $c'_1 + \pi = \frac{3}{2}x_{12}$ .

Now we can obtain the same result on the relation between extremal designs and concavity of the vertical costs as in previous sections:

**Proposition 5** *A necessary condition for an interior design in the duopoly setting with sequential decisions is that the vertical transport cost are locally convex. Moreover, if these costs are concave, then both firms choose an extremal design.*

**Proof.** In order to have an interior design decision one needs  $\frac{d^2\Pi_1(s)}{ds_1^2} \leq 0$ . Algebraic manipulation of this second order condition provides the desired result:

$$\begin{aligned} \frac{d^2\Pi_1(s)}{ds_1^2} &= 2 \frac{\partial x_{12}}{\partial s_1} \left( \frac{2}{3} (c'_1 + \pi) - x_{12} \right) + 2x_{12} \left( -\frac{2}{3} c''_1 - \frac{\partial x_{12}}{\partial s_1} \right) = \frac{4}{3} \left( \frac{\partial x_{12}}{\partial s_1} (c'_1 + \pi) - x_{12} c''_1 \right) - 4x_{12} \frac{\partial x_{12}}{\partial s_1} \\ &\stackrel{FOC}{=} 2x_{12} \frac{\partial x_{12}}{\partial s_1} + 2x_{12} \left( -\frac{2}{3} c''_1 \right) - 4x_{12} \frac{\partial x_{12}}{\partial s_1} = 2x_{12} \left( -\frac{\partial x_{12}}{\partial s_1} - \frac{2}{3} c''_1 \right) \leq 0 \end{aligned}$$

This is equivalent to

$$\begin{aligned}
\frac{\partial x_{12}}{\partial s_1} + \frac{2}{3}c_1'' &\geq 0 \\
\Leftrightarrow \frac{1}{3} \frac{c_1' + \pi}{s_1 + s_2} - \frac{x_{12}}{s_1 + s_2} + \frac{2}{3}c_1'' &\geq 0 \stackrel{FOC}{\Leftrightarrow} \frac{\frac{1}{2}x_{12}}{s_1 + s_2} - \frac{x_{12}}{s_1 + s_2} + \frac{2}{3}c_1'' \geq 0 \\
\Leftrightarrow -\frac{1}{2} \frac{x_{12}}{(s_1 + s_2)} + \frac{2}{3}c_1'' &\geq 0
\end{aligned}$$

Note that  $c_1'' = c''(1 - s_1) < 0$  is sufficient for this to fail and, so, if  $c$  is concave then, again, we have extremal designs. Again convexity is necessary (but not sufficient) for an interior solution. ■

## 5 Conclusions

- We propose a tractable model of product design. Product design as a (costless) trade-off between vertical quality and horizontal dispersion. This is the most relevant setting, if there is no trade-off, costless design is trivial. Adding design costs to our setting is straightforward.
- This trade-off results in design being represented as demand rotations. Our setting provides a simple and general framework to construct demand rotations
- We characterize sufficient conditions for design to be extremal and to be interior. These conditions apply in monopoly and a range of competitive models. In particular, a necessary condition for interior design is that consumers' vertical transport costs are locally convex; if these costs are concave then designs are necessarily extremal (either as broad or as targeted as possible)
- The results on extremal design choices provide some theoretical foundations and conditions for Porter's "stuck in the middle" problem

- We also show how lower ex-ante vertical quality pushes the firm towards more niche designs.
- Our model is an intuitive representation of design and provides a workhorse for further applications

## References

- [1] Anand, Bharat and Shachar, Ron (2011) “Advertising, the Matchmaker,” *RAND Journal of Economics*, 42 (2), 205-245.
- [2] Anderson, Simon P. and Régis Renault (1999) “Pricing, product diversity and search costs: a Bertrand-Chamberlin-Diamond model,” *RAND Journal of Economics*, Vol. 30 No. 4, Winter, pp. 719-735.
- [3] Bar-Isaac, Heski, Guillermo Caruana, and Vicente Cuñat, 2012, “Search, Design, and Market Structure,” *American Economic Review*, 102(2): 1140-1160.
- [4] Diamond, Peter (1971) “A Model of Price Adjustment,” *Journal of Economic Theory*, 3, 156-68.
- [5] Ganuza, Juan-Jose and Jose Penalva (2010) “Signal Orderings based on Dispersion and the Supply of Private Information in Auctions” *Econometrics*, 78 (3), 1007-1030.
- [6] Kamenica, Emir, and Matthew Gentzkow. 2011. “Bayesian Persuasion.” *American Economic Review*, 101(6): 2590-2615.
- [7] Kuksov (2004) “Buyer Search Costs and Endogenous Product Design,” *Marketing Science*, 23(4), 490-99.

- [8] Kuksov, Dmitri, and Yuanfang Lin. 2010. "Information provision in a vertically differentiated competitive marketplace," *Marketing Science*, 29(1): 122-138.
- [9] Larson, Nathan, 2011 "Niche Products, Generic Products, and Consumer Search," *Economic Theory*, 1-40.
- [10] Lewis, Tracy R. and Sappington, David E. M. "All-or-Nothing Information Control," *Economics Letters*, 1991, 37(2), pp. 111-13.
- [11] Lewis, Tracy R. and Sappington, David E. M. "Supplying Information to Facilitate Price Discrimination," *International Economic Review*, 1994, 35(2), pp. 309-27.
- [12] McCall, John J. 1970. "Economics of Information and Job Search," *The Quarterly Journal of Economics Inquiry*, 84(1), pp. 113-126.
- [13] Milgrom, Paul and Shannon, Chris. 1994. "Monotone Comparative Statics", *Econometrica*, Vol 61, Issue 1, 157-180.
- [14] Ottaviani, Marco, and Andrea Prat. 2001. "The Value of Public Information in Monopoly," *Econometrica*, 69(6): 1673-1683.
- [15] Porter, Michael E. 1980. *Competitive Strategy*. Free Press, New York.
- [16] Rayo, Luis, and Ilya Segal. 2010. "Optimal information disclosure," *Journal of Political Economy*, 118(5): 949-987.
- [17] Saak, Alexander E., 2006, "The optimal private information in single unit monopoly," *Economics Letters*, 91(2), pp 267-272.
- [18] Salop, Steven C., 1979, "Monopolistic competition with outside goods," *Bell Journal of Economics*, 10, 141-156.

- [19] von Ungern-Sternberg, Thomas. 1988. "Monopolistic Competition and General Purpose Products," *Review of Economic Studies* 55(2), pp. 231-246.
- [20] Wolinsky, Asher. 1986. "True Monopolistic Competition as a Result of Imperfect Information," *The Quarterly Journal of Economics*, Vol. 101 No. 3, pp. 493-511.